Math 106 Take-Home Quiz #4

Due Sunday, March 2, 2025 in Gradescope by 11:59 pm

Instructions:

• This is an Open Notes Quiz. You can use materials, homeworks problems, lecture notes, etc. that you manually worked on.

• This is **NOT** an Open Internet Quiz. You can only access our Main Course Webpage.

• You are not allowed to work on or discuss these problems with other students, professor, Math Fellow TA or simply put anyone.

• You can ask a few small, clarifying, questions in Office Hours, but the problems will not be solved for you.

• The main goal is to make a thoughtful and detailed presentation for the solutions. Submit a clear final draft. No mess please.

• Please submit your final work in Gradescope in the Quiz 4 entry.

1. [10 points] Evaluate each of the Limits. You may need to use algebra to decompose the Limit into simpler pieces. Also use arrows to show the size arguments, either growing large towards ∞ and/or small towards 0. Simplify.

(a) $\lim_{n\to\infty} \frac{50}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{100}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{66}{n}(n) =$

(b) $\lim_{n\to\infty} \frac{18}{n}(n) - \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) =$

2. [10 points] Here i and n are some constants. Simplify, combine similar variables.

(a) Consider $f(x) = 5 - 4x - 3x^2$. Compute $f\left(-2 + \frac{3i}{n}\right)$.

(b) Consider $f(x) = x^2 - 4x - 7$. Compute $f\left(-3 + \frac{4i}{n}\right)$.

3. [10 points] Show that

$$\sum_{i=1}^{n}\left[6 - 3\left(-3 + \frac{4i}{n}\right) - 2\left(-3 + \frac{4i}{n}\right)^2\right]\cdot\left(\frac{4}{n}\right) = \boxed{-12 + \frac{144}{n^2}\sum_{i=1}^{n} i - \frac{128}{n^3}\sum_{i=1}^{n} i^2}$$

DO NOT SPEAK TO ANYONE ELSE ABOUT THIS QUIZ