Quiz 3 Answer Key

1a. $$\int 4x^7 + \frac{7}{x^4} + x^{4/7} + \frac{1}{x^{7/4}} - \frac{4}{7x^{4/7}} - \frac{7}{4} + \frac{7}{4}x + \frac{1}{7x^7} - \frac{7}{4x^7}\,dx$$

prep

$$= \int 4x^7 + 7x^{-4} + x^{\overset{+7/7}{4/7}} + x^{\overset{+4/4}{-7/4}} - \frac{4}{7}x^{\overset{+7/7}{-4/7}} - \frac{7}{4} + \frac{7}{4}x + \frac{1}{7}x^{-7} - \frac{7}{4}x^{-7}\,dx$$

$$= \cancel{4}\frac{x^8}{\cancel{8}_{\,2}} + 7\frac{x^{-3}}{-3} + \frac{x^{11/7}}{11/7} + \frac{x^{-3/4}}{-3/4} - \frac{4}{\cancel{7}}\cdot\frac{x^{3/7}}{3/7} - \frac{7}{4}\cdot x + \frac{7}{4}\frac{x^2}{2} + \frac{1}{7}\frac{x^{-6}}{-6} - \frac{7}{4}\frac{x^{-6}}{-6} + C$$

$$= \boxed{\frac{x^8}{2} - \frac{7}{3x^3} + \frac{7}{11}x^{11/7} - \frac{4}{3x^{3/4}} - \frac{4}{3}x^{3/7} - \frac{7}{4}x + \frac{7x^2}{8} - \frac{1}{42x^6} + \frac{7}{24x^6} + C}$$

1b. $$\int (x+1)(x+2)\,dx = \int x^2 + 2x + x + 2\,dx = \int x^2 + 3x + 2\,dx$$

$$= \boxed{\frac{x^3}{3} + \frac{3x^2}{2} + 2x + C}$$

1c. $$\int \frac{\left(x^2 + \frac{1}{x}\right)\left(x + \frac{1}{x^2}\right)}{\sqrt{x}}\,dx = \int \frac{x^3 + \cancel{\frac{x^2}{x^2}}^{\,1} + \cancel{\frac{x}{x}}^{\,1} + \frac{1}{x^3}}{\sqrt{x}}\,dx$$

$$= \int \frac{x^3 + 2 + x^{-3}}{x^{1/2}}\,dx$$ split-split

$$= \int \frac{x^3}{x^{1/2}} + \frac{2}{x^{1/2}} + \frac{x^{-3}}{x^{1/2}}\,dx$$

prep

Algebra $$= \int x^{6/2-1/2} + 2x^{-1/2} + x^{-6/2-1/2}\,dx$$

$$= \int x^{5/2} + 2x^{-1/2} + x^{\overset{+2/2}{-7/2}}\,dx$$

$$= \frac{x^{7/2}}{7/2} + 2\frac{x^{1/2}}{1/2} + \frac{x^{-5/2}}{-5/2} + C$$

$$= \boxed{\frac{2}{7}x^{7/2} + 4x^{1/2} - \frac{2}{5x^{5/2}} + C}$$

1d. $\int \sec^2 x + 2\sin x - \cos x - \sec x \cdot \tan x \, dx = \tan x + 2(-\cos x) - \sin x - \sec x + C$

$$= \boxed{\tan x - 2\cos x - \sin x - \sec x + C}$$

1e. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)(1+x)\,dx \overset{\text{FOIL}}{=} \int \sqrt{x} + x\sqrt{x} + \frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} \, dx$

$$= \int x^{1/2} + x^{3/2} + x^{-1/2} + x^{1/2}\,dx$$

$$= \int 2x^{1/2} + x^{3/2} + x^{-1/2}\,dx$$

$$= 2\frac{x^{3/2}}{3/2} + \frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} + C = \boxed{\frac{4}{3}x^{3/2} + \frac{2}{5}x^{5/2} + 2\sqrt{x} + C}$$

2a. $f''(x) = -3 + 12x - 12x^2$

Antidifferentiate $f'(x) = \int f''(x)\,dx = \int -3 + 12x - 12x^2\,dx$

$$= -3x + \cancel{12}^{6} \cdot \frac{x^2}{\cancel{2}} - \cancel{12}^{4}\frac{x^3}{\cancel{3}} + C$$

$$= -3x + 6x^2 - 4x^3 + C$$

Use given value to solve for +C

$$f'(1) = \underbrace{-3+6}_{3}\underbrace{-4}_{-1} + C \overset{\text{set}}{=} -4$$

$$-1 + C = -4 \Rightarrow C = -3$$

$$+1 \qquad +1$$

Collect $f'(x) = -3x + 6x^2 - 4x^3 - 3$

Antidifferentiate Again $f(x) = \int f'(x)\,dx = \int -3x + 6x^2 - 4x^3 - 3\,dx$

new constant

$$= -\frac{3x^2}{2} + \cancel{6}^{2}\frac{x^3}{\cancel{3}} - \cancel{4}\frac{x^4}{\cancel{4}} - 3x + D$$

$$= -\frac{3x^2}{2} + 2x^3 - x^4 - 3x + D$$

Use given value to solve for +D

$$f(0) = 0 + 0 - 0 - 0 + D \overset{\text{set}}{=} 4 \Rightarrow D = 4$$

Collect

$$f(x) = \boxed{-\frac{3x^2}{2} + 2x^3 - x^4 - 3x + 4}$$

2b. Finally, $f(1) = -\frac{3}{2} + \underbrace{2-1}_{1} \underbrace{- 3 + 4}_{1} = -\frac{3}{2} + 2 = -\frac{3}{2} + \frac{4}{2} = \boxed{\frac{1}{2}}$

3a. $f'(x) = \sec^2 x - 4\sin x$

$$f(x) = \int f'(x)\,dx = \int \sec^2 x - 4\sin x\,dx$$

$$= \tan x + 4\cos x + C$$

Use given value to solve for $+C$

$$f(\pi) = \tan\pi + 4\cos\pi + C \overset{\text{set}}{=} -6$$

$$= \frac{\overset{0}{\sin\pi}}{\underset{-1}{\cos\pi}} + 4\overset{-1}{\cos\pi} + C = -6$$

$$\underset{+4}{0 - 4} + C = \underset{+4}{-6} \Rightarrow C = -2$$

Collect $f(x) = \boxed{\tan x + 4\cos x - 2}$

3b. $f\left(\frac{\pi}{3}\right) = \overset{\sqrt{3}}{\tan\frac{\pi}{3}} + 4\overset{1/2}{\cos\frac{\pi}{3}} - 2 = \sqrt{3} + \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right) \overset{0}{- 2} = \sqrt{3} + \cancel{2} - \cancel{2} = \boxed{\sqrt{3}}$