Quiz #2 Answer Key

1(a) $f(x)=\sqrt{\cos\left(3x^2-\frac{1}{x^8}\right)}$ prep $-x^{-8}$

$$f'(x)=\frac{1}{2\sqrt{\cos\left(3x^2-\frac{1}{x^8}\right)}}\cdot\left(-\sin\left(3x^2-\frac{1}{x^8}\right)\right)\cdot\left(6x+8x^{-9}\right)$$

1(b) $f(x)=\sin(x^2)\cdot\tan(3x)$

$$f'(x)=\sin(x^2)\cdot\sec^2(3x)\cdot(3)+\tan(3x)\cdot\cos(x^2)\cdot(2x)$$

1(c) $f(x)=\sin^2x+\cos^2x=(\sin x)^2+(\cos x)^2$

$f'(x)=2\sin x\cdot\cos x+2\cos x(-\sin x)=2\sin x\cos x-2\sin x\cos x=0$ Match!

OR Note: $f(x)=\sin^2x+\cos^2x=1$ Constant $\Rightarrow f'(x)=0$ for free

2. $f(x)=\frac{\sin x}{1+\cos x}$

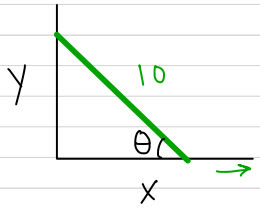
$$f'(x)=\frac{(1+\cos x)\cdot\cos x-\sin x\cdot(-\sin x)}{(1+\cos x)^2}=\frac{\cos x+\cos^2x+\sin^2x}{(1+\cos x)^2}$$

$$=\frac{\cos x+1}{(1+\cos x)^2}=\frac{1}{1+\cos x}$$

$$f'\left(\frac{\pi}{3}\right)=\frac{1}{1+\cos\left(\frac{\pi}{3}\right)}=\frac{1}{3/2}=\frac{2}{3}$$

($\cos\frac{\pi}{3}=\frac{1}{2}$)

3. Diagram



Variables

Let x = Distance from Bottom of Ladder to Wall

y = Distance from Top of Ladder to Ground

θ = Angle Formed between Ladder and Ground

Given $\frac{dy}{dt} = -1 \text{ ft/sec}$ (Decreasing y)

Find $\frac{d\theta}{dt} = ?$ when $y = 5$

Equation

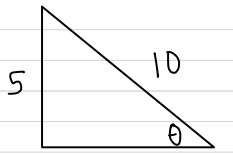
$$\sin\theta = \frac{y}{10}$$

Differentiate

$$\frac{d}{dt}\left(\sin\theta\right) = \frac{d}{dt}\left(\frac{y}{10}\right)$$

$$\cos\theta \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

Extra Solvable Information



$\Rightarrow \sqrt{10^2 - 5^2} = \sqrt{100-25} = \sqrt{75}$ (OR) $= \sqrt{25}\sqrt{3} = 5\sqrt{3}$

↳ $\cos\theta = \frac{\sqrt{75}}{10}$ $\frac{A}{H}$

Substitute Key Moment Info

$$\frac{\sqrt{75}}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10}(-1) \quad \frac{10}{\sqrt{75}}$$

Solve $\frac{d\theta}{dt} = -\frac{1}{\sqrt{75}}$ Radians/Second

Answer

The angle between the ladder and the ground is decreasing at a rate of $\frac{1}{\sqrt{75}}$ Radians per Second. at that moment.

4. Let $f(x) = \sin x + \cos(2x)$

$f'(x) = \cos x - 2\sin(2x)$

$f'(\pi/6) = \cos\left(\frac{\pi}{6}\right) - 2\sin\left(2 \cdot \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

($\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$)

Common Denominator