1. $f(x) = 4x^3 - 3x^4 - \dfrac{3}{4} + \dfrac{4}{x^3} - \dfrac{3}{x^4} - 3x - x^{\frac{4}{3}} + x^{\frac{3}{4}} + \dfrac{1}{x^{\frac{3}{4}}}$ ↑

$\overset{prep}{=} 4x^3 - 3x^4 - \dfrac{3}{4} + 4x^{-3} - 3x^{-4} - 3x - x^{\frac{4}{3}} + x^{\frac{3}{4}} + x^{-\frac{3}{4}}$

Constant

$$f'(x) = \boxed{12x^2 - 12x^3 - 0 - 12x^{-4} + 12x^{-5} - 3 - \dfrac{4}{3}x^{\frac{1}{3}} + \dfrac{3}{4}x^{-\frac{1}{4}} - \dfrac{3}{4}x^{-\frac{7}{4}}}$$

2. $f(x) = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right) \cdot \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$ · FOIL Algebra 1st to simplify, then Derivative

FOIL
$= \sqrt{x}\cdot\sqrt{x} + \sqrt{x}\cdot\dfrac{1}{\sqrt{x}} + \sqrt{x}\cdot\dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{x}}\cdot\dfrac{1}{\sqrt{x}}$

$= x + 1 + 1 + \dfrac{1}{x} = x + 2 + x^{-1}$

$f'(x) = 1 - x^{-2} = \boxed{1 - \dfrac{1}{x^2}}$ Match!

OR Product Rule 1st, Algebra 2nd. Messier $\dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \rightarrow -\dfrac{1}{2}x^{-\frac{3}{2}}$

$f(x) = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right) \cdot \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$

$f'(x) = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)\cdot\left(\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}x^{-\frac{3}{2}}\right) + \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)\cdot\left(\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}x^{-\frac{3}{2}}\right)$

Same pieces in Product Rule

FOIL
$= \dfrac{\sqrt{x}}{2\sqrt{x}} - \dfrac{1}{2}x^{\frac{1}{2}}x^{-\frac{3}{2}} + \dfrac{1}{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}x^{-\frac{1}{2}}x^{-\frac{3}{2}} + \dfrac{\sqrt{x}}{2\sqrt{x}} - \dfrac{1}{2}x^{\frac{1}{2}}x^{-\frac{3}{2}} + \dfrac{1}{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2}x^{-\frac{1}{2}}x^{-\frac{3}{2}}$

$= \dfrac{1}{2} - \dfrac{1}{2}x^{-1} + \dfrac{1}{2x} - \dfrac{1}{2}x^{-2} + \dfrac{1}{2} - \dfrac{1}{2}x^{-1} + \dfrac{1}{2x} - \dfrac{1}{2}x^{-2}$

$= \boxed{1 - \dfrac{1}{x^2}}$ Match!

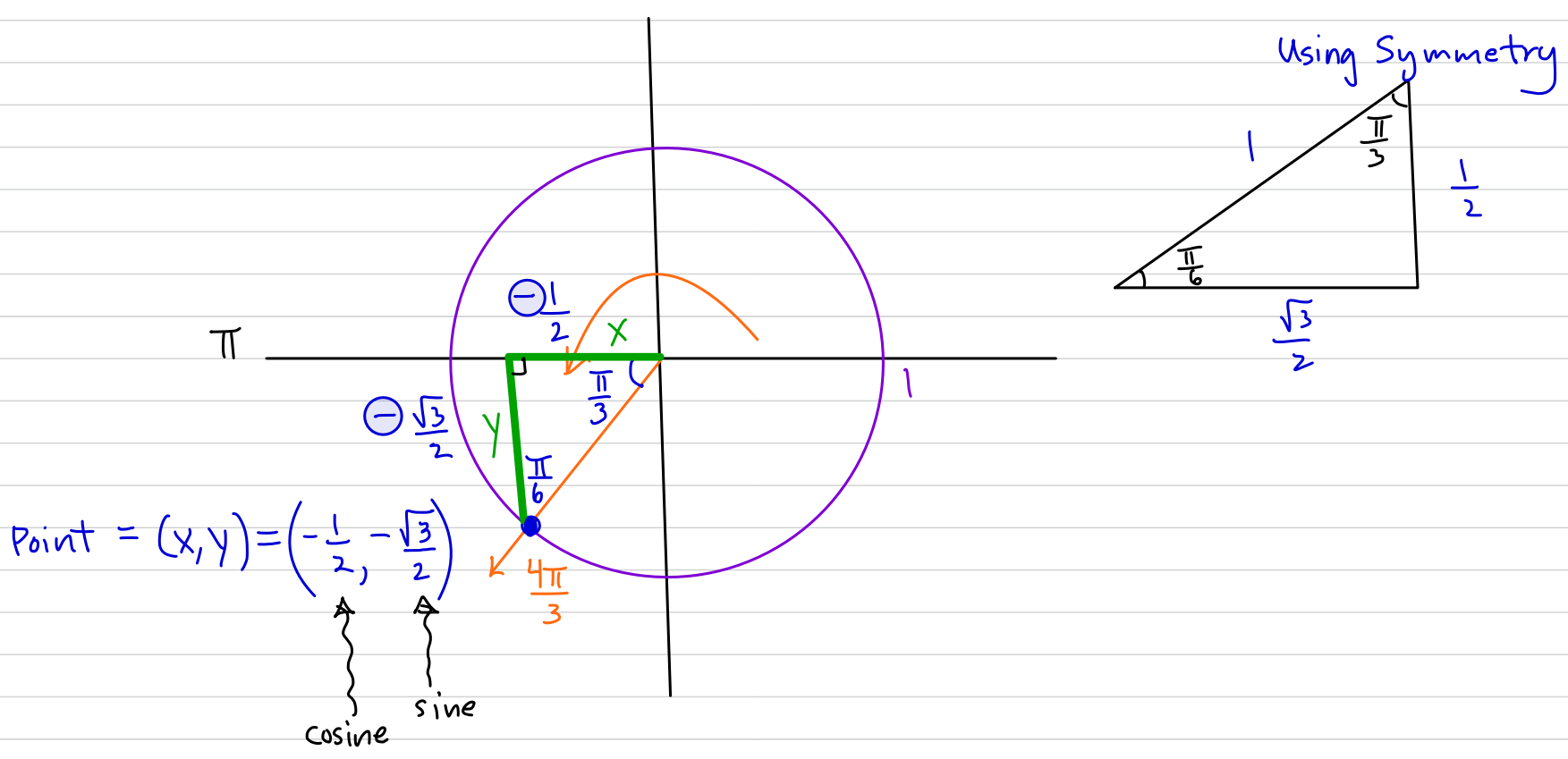3. $f(x) = \dfrac{x^3 - 7x}{5 - 4x^2}$ Quotient Rule

$$f'(x) = \frac{(5-4x^2)(3x^2-7) - (x^3-7x)(-8x)}{(5-4x^2)^2} = \frac{15x^2 - 35 - 12x^4 + 28x^2 + 8x^4 - 56x^2}{(5-4x^2)^2}$$

$$= \boxed{\frac{-4x^4 - 13x^2 - 35}{(5-4x^2)^2}} \quad \text{Match!}$$

4. $f(x) = \sqrt{5x^4 - 7 - \dfrac{1}{x^3}\, x^{-3}}$

$$\boxed{f'(x) = \frac{1}{2\sqrt{5x^4 - 7 - x^{-3}}} \cdot \left(20x^3 + 3x^{-4}\right)}$$

5. $\cos\left(\dfrac{4\pi}{3}\right) = \boxed{-\dfrac{1}{2}}$ $\sin\left(\dfrac{4\pi}{3}\right) = \boxed{-\dfrac{\sqrt{3}}{2}}$

Using Symmetry

$$\text{Point} = (x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

cosine   sine

Note: Angle $\dfrac{4\pi}{3}$ is in Quadrant 3 where BOTH x and y are Negative