Math 106, Spring 2024

Homework #5

Due **Wednesday, February 14th** in Gradescope by 11:59 pm ET

Goal: More Related Rates with Trigonometry and Starting Antiderivatives

FIRST: Read through and understand the following Examples.

Compute the following Antiderivatives:

Ex: $\displaystyle\int x^3 + \frac{1}{x^6} + 7\ dx \overset{\text{prep}}{=} \int x^3 + x^{-6} + 7\ dx = \frac{x^4}{4} + \frac{x^{-5}}{-5} + 7x + C = \boxed{\frac{x^4}{4} - \frac{1}{5x^5} + 7x + C}$

Ex: $\displaystyle\int 5x^2 - \frac{6}{x^4}\ dx \overset{\text{prep}}{=} \int 5x^2 - 6x^{-4}\ dx = 5\left(\frac{x^3}{3}\right) - 6\left(\frac{x^{-3}}{-3}\right) + C = \boxed{\frac{5x^3}{3} + \frac{2}{x^3} + C}$

Ex: $\displaystyle\int (x^3 + 3x)\left(\frac{1}{x^3} - 5x^6\right)\ dx \overset{\text{FOIL}}{=} \int 1 - 5x^9 + 3x^{-2} - 15x^7\ dx$

$$= x - 5\left(\frac{x^{10}}{10}\right) + 3\left(\frac{x^{-1}}{-1}\right) - 15\left(\frac{x^8}{8}\right) + C$$

$$= \boxed{x - \frac{x^{10}}{2} - \frac{3}{x} - \frac{15x^8}{8} + C}$$

Ex: $\displaystyle\int \frac{5 + x^3}{x^7}\ dx \overset{\text{split}}{=} \int \frac{5}{x^7} + \frac{x^3}{x^7}\ dx \overset{\text{prep}}{=} \int 5x^{-7} + x^{-4}\ dx = 5\left(\frac{x^{-6}}{-6}\right) + \frac{x^{-3}}{-3} + C = \boxed{-\frac{5}{6x^6} - \frac{1}{3x^3} + C}$

Ex: Consider $f(x)$ where $f'(x) = \dfrac{1}{\sqrt{x}} - \dfrac{1}{x^3}$ and $f(4) = 2$. Find $f(x)$.

$$f(x) = \int f'(x)\ dx = \int \frac{1}{\sqrt{x}} - \frac{1}{x^3} \overset{\text{prep}}{=} \int x^{-\frac{1}{2}} - x^{-3}\ dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{-2}}{-2} + C = 2\sqrt{x} + \frac{1}{2x^2} + C$$

Use the Initial Condition to solve for $+C$.

$$f(4) = 2\sqrt{4}^{2} + \frac{1}{2(4)^2} + C = 4 + \frac{1}{32} + C \overset{\text{set}}{=} 2$$

Solve for $+C$: $C = 2 - 4 - \dfrac{1}{32} = -2 - \dfrac{1}{32} = -\dfrac{64}{32} - \dfrac{1}{32} = -\dfrac{65}{32}$

Finally, piecing this together, we have $f(x) = \boxed{2\sqrt{x} + \dfrac{1}{2x^2} - \dfrac{65}{32}}$

Next, Complete the following Homework problems.

1. A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

Compute the following Indefinite Integrals in order to find the Most General Antiderivative of each function.

2. $\int x - 3 \; dx$

3. $\int 5x^9 - 3x^6 + 12x^3 \; dx$

4. $\int 7 + \frac{3}{4}x^2 - \frac{4}{5}x^3 \; dx$

5. $\int 7x^{\frac{2}{5}} + 8x^{-\frac{4}{5}} + \sqrt{2} \; dx$

6 $\int \frac{10}{x^9} + \frac{9}{x^4} \; dx$

7. $\int \frac{1}{x^{\frac{2}{7}}} + \frac{1}{2\sqrt{x}} \; dx$

8. $\int x^2 - \frac{5}{x^3} + \frac{2}{3}x^{\frac{2}{3}} \; dx$

9. $\int \frac{1 + x^2 + x^9}{x^2} \; dx$

10. $\int 2\sin x - 7\sec^2 x - 3\sec x \tan x \; dx$

11. $\int (x+1)(2x-1) \; dx$

Find the function f which satisfies each of the following:

12. $f'(x) = 1 + 3\sqrt{x}$ and $f(4) = 25$

13. $f'(x) = \sin x$ and $f(\pi) = -5$

14. $f''(x) = \sin x + \cos x$ and $f'(0) = 4$ and $f(0) = 3$

15. $f''(x) = 20x^3 - 12x^2 + 6x$ and $f'(1) = -5$ and $f(1) = -10$.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Alexa, SMUDD 208A

Wednesday: 1:00-3:00 pm

Thursday: none for Professor

6:00–7:30 pm TA Alexa, SMUDD 208A

Friday: 12:00–2:00 pm

- Please take the time to read over your class notes this week.
- Try to understand the Trig concepts and not just the numbers and formulas.