



Math 106 Exam 3

April 24, 2026



- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [12 Points] (a) Compute the Derivative y' where $y = \ln\left(\frac{\sqrt{9-x^3} \cdot e^{\cos x}}{(1-e^x)^7 \cdot (\ln x)}\right)$

Do not simplify your final answer here.

(b) Solve for all x such that $\ln(6-5x) = 4$. Simplify

2. [8 Points] Compute $\frac{dy}{dx}$ where $y = (\cos x)^{\ln x}$

3. [8 Points] Find the Local Maximum and Minimum **Value(s)** for $f(x) = \frac{x^2}{e^x}$

4. [12 Points]

(a) Consider $f(x) = \cos(e^{5x} - 1) + \ln 1 + \sin(\ln(1 + 6x))$ Compute $f'(0)$.

(b) Consider $f(x) = e^{\sin(3x)} - \ln(e^7) - \ln(1 + \sin(5x))$ Compute $f'(0)$.

5. [12 Points]

(a) Consider

$$f(x) = 8 \ln x - \frac{8}{e^{8x}} + e^{\ln 8} - \frac{8}{x} + \ln 8 + \frac{e^x}{e^{8x}} \quad \text{Compute } f'(x).$$

(b) Consider a *different* equation

$$g(x) = 8e^x + \frac{1}{e^x} - \ln(e^x) - \frac{1}{x^8} - \frac{1}{x} + (e^{8x} \cdot e^x) \quad \text{Compute } \int g(x) dx.$$

6. [24 Points] Compute the following Indefinite Integrals.

(a) $\int \frac{6 - x^7}{x^8} dx$

(b) $\int \frac{x^7}{6 - x^8} dx$

(c) $\int \frac{(1 + e^{3x})^2}{e^{3x}} dx$

(d) $\int \frac{e^{3x}}{(1 + e^{3x})^2} dx$

7. [24 Points] Compute each of the following Definite Integrals. Simplify. Justify all steps.

(a) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \, dx \stackrel{\text{hint}}{=} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \, dx = \boxed{\frac{1}{2} \ln 3}$

(b) Show that $\int_{e^2}^{e^7} \frac{8}{x\sqrt{2 + \ln x}} \, dx = \boxed{16}$

(c) Show that $\int_0^{\ln 3} \frac{1}{e^{2x}(1 + e^{-2x})^2} \, dx = \boxed{\frac{1}{5}}$

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1

Compute $\lim_{n \rightarrow \infty} \frac{2 \left[e^{(3+\frac{2}{n})} + e^{(3+\frac{4}{n})} + e^{(3+\frac{6}{n})} + \dots + e^{(3+\frac{2(n-1)}{n})} + e^5 \right]}{n}$

OPTIONAL BONUS #2 Compute $\int \frac{1}{1 - \sin x} \, dx$