Math 106 Exam 3
April 25, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [12 Points] Compute the Derivative y' where $y = \ln\left(\dfrac{(9-x^5)^7 \cdot e^{\cos x}}{\sqrt{8-x^3}\cdot \ln(\ln x)}\right)$

Do not simplify your final answer here.

2. [10 Points] Compute $\dfrac{dy}{dx}$ where $y = (\cos x)^{\sqrt{x}}$

3. [16 Points]

(a) Let $f(x) = \sin\left(\ln(1+7x)\right) - \ln\left(1+\cos(4x)\right)$ Compute $f'(0)$.

(b) Let $f(x) = \dfrac{e^{8x}}{e^{3x}} - e^{\sin(6x)} - \cos\left(1-e^{8x}\right)$ Compute $f'(0)$.

4. [12 Points] Consider $f(x) = x^2e^x$

Find all of the Local Maximum and Minimum **Values**.

5. [14 Points] Compute the following Indefinite Integrals.

(a) $\displaystyle\int \left(e^{5x} + \frac{1}{e^{2x}}\right)\left(e^x - \frac{1}{e^{7x}}\right)\,dx$

(b) $\displaystyle\int \frac{7x}{x^2+1}\,dx$

6. [36 Points] Evaluate each of the following Definite Integrals. Simplify. Justify all steps.

(a) Show that $\displaystyle\int_0^{\frac{\pi}{3}} \tan x \; dx = \boxed{\ln 2}$

(b) Show that $\displaystyle\int_{e^2}^{e^9} \frac{1}{x\sqrt{7+\ln x}} \; dx = \boxed{2}$

(c) Show that $\displaystyle\int_0^{\ln 3} \frac{1}{e^{2x}\left(1+e^{-2x}\right)^2} \; dx = \boxed{\frac{1}{5}}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\displaystyle\int \frac{(\ln(\ln x))^2}{x \ln x \left[5+\ln(\ln x)\right]^2} \; dx$