Math 106 Exam 3
April 26, 2024

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $\ln(e^3)$, $e^{2\ln 3}$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points] Compute the following Derivative y' where $y = \ln\left(\dfrac{\sqrt{1+e^x}\cdot\ln(3x)}{e^{\tan x}\ (4-x^5)^6}\right)$

Do not simplify your final answer here.

2. [10 Points] Compute $\dfrac{dy}{dx}$ where $y = (\cos x)^x$

3. [12 Points]

(a) Consider $f(x) = \ln(1+\cos x) - e^{\sin x} - \ln 5$ Compute $f'(0)$.

(b) Consider $f(x) = \cos(\ln(1+x)) - e^x + e^{(e^x)} + \dfrac{e}{1+\ln(x+1)}$ Compute $f'(0)$.

4. [12 Points]

(a) Consider $f(x) = e^{8x} + \dfrac{8}{e^{8x}} + e^{\ln 8} - \dfrac{8}{x} + \dfrac{e^x}{e^{8x}}$ Compute $f'(x)$.

(b) Consider $f(x) = 8e^x + \dfrac{8}{e^x} - \ln\left(e^8\right) - \dfrac{e}{x^8} + \left(e^{8x}\right)\cdot\left(e^x\right)$ Compute $\displaystyle\int f(x)\,dx$

5. [26 Points] Compute the following Indefinite Integrals.

(a) $\displaystyle\int \frac{(1+e^{3x})^2}{e^{3x}}\, dx$

(b) $\displaystyle\int \frac{e^{3x}}{(1+e^{3x})^2}\, dx$

(c) $\displaystyle\int \frac{1}{\sqrt{x}\; e^{\sqrt{x}}}\, dx$

(d) $\displaystyle\int \frac{5-x^8}{x^9}\, dx$

(e) $\displaystyle\int \frac{x^8}{5-x^9}\, dx$

6. [30 Points] Evaluate each of the following Definite Integrals. Simplify. Justify all steps.

(a) Show that $\displaystyle\int_0^{\frac{\pi}{3}} \tan x\; dx = \boxed{\ln 2}$

(b) Show that $\displaystyle\int_0^{\ln 2} \frac{e^{3x}}{\sqrt{8+e^{3x}}}\, dx = \boxed{\frac{2}{3}}$

(c) Show that $\displaystyle\int_e^{e^5} \frac{1}{x(3+\ln x)}\, dx = \boxed{\ln 2}$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\displaystyle\lim_{n\to\infty} \frac{e^{(1+\frac{1}{n})} + e^{(1+\frac{2}{n})} + e^{(1+\frac{3}{n})} + \ldots + e^2}{n}$