Exam 2 Spring 2026 Answer Key

1 Limit Definition only this time

$$\int_{-1}^{2} 2-2x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\cdot\left(\frac{3}{n}\right)$$

$a=-1$
$b=2$

$$\Delta x=\frac{b-a}{n}=\frac{2-(-1)}{n}=\frac{3}{n}$$

$$x_i = a+i\Delta x = -1+i\left(\frac{3}{n}\right) = -1+\frac{3i}{n}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 2-2\left(-1+\frac{3i}{n}\right)-\left(-1+\frac{3i}{n}\right)^2$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 2+2-\frac{6i}{n}-\left(1-\frac{6i}{n}+\frac{9i^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 2+2-\cancel{\frac{6i}{n}}-1+\cancel{\frac{6i}{n}}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 3-\frac{9i^2}{n^2}$$

pull all non-i values out of Series Sums

$$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n} 3-\frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{9}{n}\overset{n}{\cancel{\sum_{i=1}^{n}1}}-\frac{27}{n^3}\sum_{i=1}^{n} i^2$$

i-Formula

Note: no i terms this time

repartner

$$= \lim_{n\to\infty}\frac{9}{\cancel{n}}\cdot\cancel{n}-\frac{27}{\underset{n\cdot n\cdot n}{n^3}}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} 9-\frac{27}{6}\left(\overset{1}{\cancel{\frac{n}{n}}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty} 9-\frac{27}{6}(1)\left(\overset{1}{\cancel{\frac{n}{n}}}+\frac{1}{n}\right)\left(\overset{2}{\cancel{\frac{2n}{n}}}+\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 9-\frac{27}{6}\left(1+\overset{0}{\cancel{\frac{1}{n}}}\right)\left(2+\overset{0}{\cancel{\frac{1}{n}}}\right)$$

Finally ⇝ Let $n\to\infty$

$$= 9-\frac{27}{\underset{3}{\cancel{6}}}(1)(\cancel{2}) = 9-9 = \boxed{0}$$

Optional Check

$$\text{FTC}\int_{-1}^{2} 2-2x-x^2\,dx = 2x-\overset{-x^2}{\frac{\cancel{2}x^2}{\cancel{2}}}-\frac{x^3}{3}\Bigg|_{-1}^{2} = \cancel{4}-\cancel{4}-\frac{8}{3}-\left(-2-1+\frac{1}{3}\right)$$

$$= -\frac{8}{3}+3-\frac{1}{3} = -\frac{9}{3}+3 = -\cancel{3}+\cancel{3} = \boxed{0}$$

2(a) $\int \frac{x^6}{\underbrace{(5-x^7)}_{u}{}^2}\,dx = -\frac{1}{7}\int \overset{u^{-2}}{\frac{1}{u^2}}\,du = \cancel{-}\frac{1}{7}\left(\frac{u^{-1}}{\cancel{-}1}\right)+C = \boxed{\frac{1}{7(5-x^7)}+C}$

$$\boxed{\begin{aligned} u&=5-x^7\\ du&=-7x^6\,dx\\ -\tfrac{1}{7}\,du&=x^6\,dx\end{aligned}}$$

u-sub

2(b) $\int \frac{(5-x^7)^2}{x^9}\,dx \underset{\substack{\text{FOIL}\\ \text{Algebra}}}{=} \int \frac{25-10x^7+x^{14}}{x^9}\,dx \underset{\text{split}}{=} \int \frac{25}{x^9}-\frac{10x^7}{x^9}+\frac{x^{14}}{x^9}\,dx$

Note: u-sub does not match

~~$u=5-x^7$~~

~~$du=-7x^6dx$~~

$= \int 25x^{-9}-10x^{-2}+x^5\,dx = 25\left(\frac{x^{-8}}{-8}\right)\cancel{-}10\left(\frac{x^{-1}}{\cancel{-}1}\right)+\frac{x^6}{6}+C$

$= \boxed{\frac{-25}{8x^8}+\frac{10}{x}+\frac{x^6}{6}+C}$

2(c) $\int \sin(3x)\cos(3x)\,dx = \frac{1}{3}\int u\,du = \frac{1}{3}\cdot\frac{u^2}{2}+C = \boxed{\frac{1}{6}\sin^2(3x)+C}$

$$\boxed{\begin{aligned} u&=\sin(3x)\\ du&=3\cos(3x)\,dx\\ \tfrac{1}{3}\,du&=\cos(3x)\,dx\end{aligned}}$$

Note: you could instead choose $u=\cos(3x)$ but you would have an extra minus sign

2(d) $\int x(x+4)^6\,dx = \int (u-4)\cdot u^6\,du = \int u^7-4u^6\,du$

$$\boxed{\begin{aligned} &u=x+4 \Rightarrow x=u-4\\ &du=dx\end{aligned}}$$

$= \frac{u^8}{8}-4\left(\frac{u^7}{7}\right)+C = \boxed{\frac{(x+4)^8}{8}-\frac{4}{7}(x+4)^7+C}$

2(e) $\int \frac{1}{x^3}\cos\left(6+\underbrace{\frac{1}{x^2}}_{u}\right)dx = -\frac{1}{2}\int \cos u\,du = -\frac{1}{2}\sin u+C = \boxed{-\frac{1}{2}\sin\left(6+\frac{1}{x^2}\right)+C}$

$$\boxed{\begin{aligned} u&=6+\frac{1}{x^2}\\ du&=-\frac{2}{x^3}\,dx\\ -\tfrac{1}{2}\,du&=\frac{1}{x^3}\,dx\end{aligned}}$$

3(a) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^5 x}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^5}\,dx = -\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \overset{u^{-5}}{\frac{1}{u^5}}\,du = \cancel{-}\left(\frac{u^{-4}}{\cancel{-}4}\right)\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} = \frac{1}{4u^4}\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$x = \frac{\pi}{6} \Rightarrow u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$x = \frac{\pi}{3} \Rightarrow u = \cos\frac{\pi}{3} = \frac{1}{2}$

$$= \frac{1}{\cancel{4}\left(\frac{1}{2}\right)^4} - \frac{1}{\cancel{4}\left(\frac{\sqrt{3}}{2}\right)^4}$$

$\cancel{\frac{1}{16}}$ 4 $\qquad$ $\cancel{\frac{9}{16}}$ 4

$$= 4 - \frac{4}{9} = \frac{36}{9} - \frac{4}{9} = \boxed{\frac{32}{9}}$$

3(b) $$\int_0^{\frac{\pi}{3}} \sin(5x)\,dx = \frac{1}{5}\int_0^{\frac{5\pi}{3}} \sin u\,du = -\frac{1}{5}\cos u\Bigg|_0^{\frac{5\pi}{3}} = -\frac{1}{5}\left(\overset{+\frac{1}{2}}{\cancel{\cos\frac{5\pi}{3}}} - \overset{1}{\cancel{\cos 0}}\right)$$

$$= -\frac{1}{5}\left(\frac{1}{2} - 1\right) = -\frac{1}{5}\cdot\left(-\frac{1}{2}\right) = \boxed{\frac{1}{10}}$$

$u = 5x$
$du = 5\,dx$
$\frac{1}{5}\,du = dx$

$x = 0 \Rightarrow u = 0$
$x = \frac{\pi}{3} \Rightarrow u = \frac{5\pi}{3}$

3(c) $$\int_9^{64} \frac{1}{\sqrt{x}\,\sqrt{1+\sqrt{x}}}\,dx = 2\int_4^9 \frac{1}{\sqrt{u}}\,du = 2\int_4^9 u^{-\frac{1}{2}}\,du = 2\left(\frac{u^{\frac{1}{2}}}{\underset{2}{\frac{1}{2}}}\right)\Bigg|_4^9$$

$u = 1+\sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\,dx$
$2du = \frac{1}{\sqrt{x}}\,dx$

$x = 9 \Rightarrow u = 1 + \overset{3}{\cancel{\sqrt{9}}} = 4$
$x = 64 \Rightarrow u = 1 + \overset{8}{\cancel{\sqrt{64}}} = 9$

$$= 4\sqrt{u}\Bigg|_4^9 = 4\left(\overset{3}{\cancel{\sqrt{9}}} - \overset{2}{\cancel{\sqrt{4}}}\right) = 4(3-2) = \boxed{4}$$

3(d) $$\int_0^5 \left|x^2-4\right|dx = \int_0^2 \overset{\substack{x<2 \\ 4-x^2 \\ -(x^2-4)}}{\cancel{|x^2-4|}}\,dx + \int_2^5 \overset{\substack{x\geq 2 \\ x^2-4}}{\cancel{|x^2-4|}}\,dx$$

See below for cases

$$= 4x - \frac{x^3}{3}\Bigg|_0^2 + \frac{x^3}{3} - 4x\Bigg|_2^5$$

$$= 8 - \frac{8}{3} - (0-0) + \frac{125}{3} - 20 - \left(\frac{8}{3} - 8\right)$$

$$\begin{array}{r} 125 \\ -16 \\ \hline 109 \end{array}$$

$$= 8 - \frac{8}{3} + \frac{125}{3} - 20 - \frac{8}{3} + 8$$

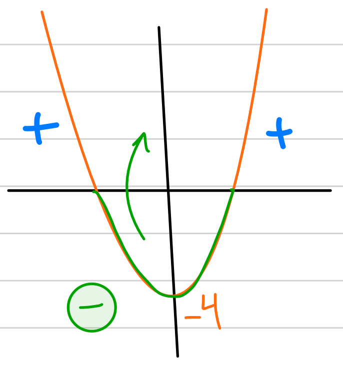
$$= -4 + \frac{109}{3} = \frac{109}{3} - \frac{12}{3} = \boxed{\frac{97}{3}}$$

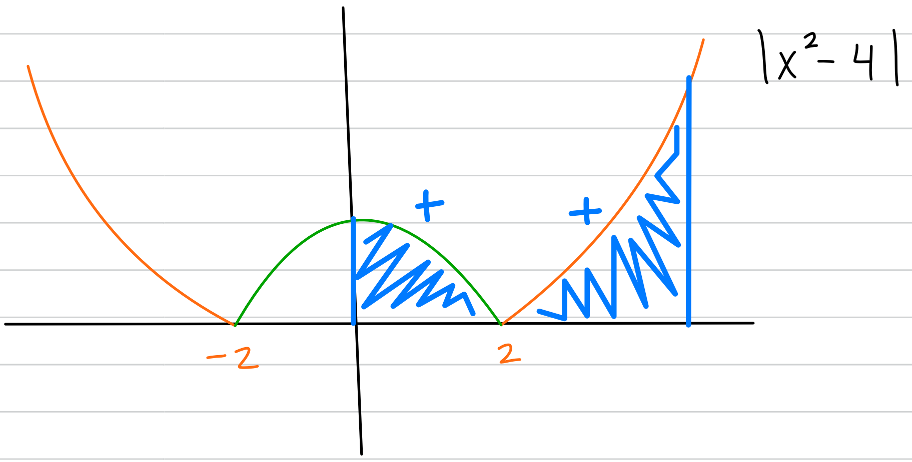
(+)

3(d) Continued

$x^2-4=0 \Rightarrow x^2=4 \quad x=\pm 2$

Optional Graph $|x^2-4|$





Cases $|x^2-4| = \begin{cases} x^2-4 & \text{if } x\le -2 \text{ or } x\ge 2 \\ -(x^2-4) & \text{if } -2<x<2 \end{cases}$

4. $f(x)=\int f'(x)\,dx = \int \frac{\sec^2 x}{\tan^3 x}\,dx = \int \frac{\sec^2 x}{\underbrace{(\tan x)}_{u}^3}\,dx = \int \frac{1}{u^3}\,\overset{\text{prep}}{du} = \int u^{-3}\,du$

$\boxed{\begin{array}{l} u=\tan x \\ du=\sec^2 x\,dx \end{array}}$

$= \frac{u^{-2}}{-2}+C = \frac{-1}{2u^2}+C = -\frac{1}{2\tan^2 x}+C$

Test value

$f\left(\frac{\pi}{3}\right) = -\frac{1}{2\left(\underbrace{\tan\left(\frac{\pi}{3}\right)}_{\sqrt{3}}\right)^2} + C \overset{\text{set}}{=} -5$

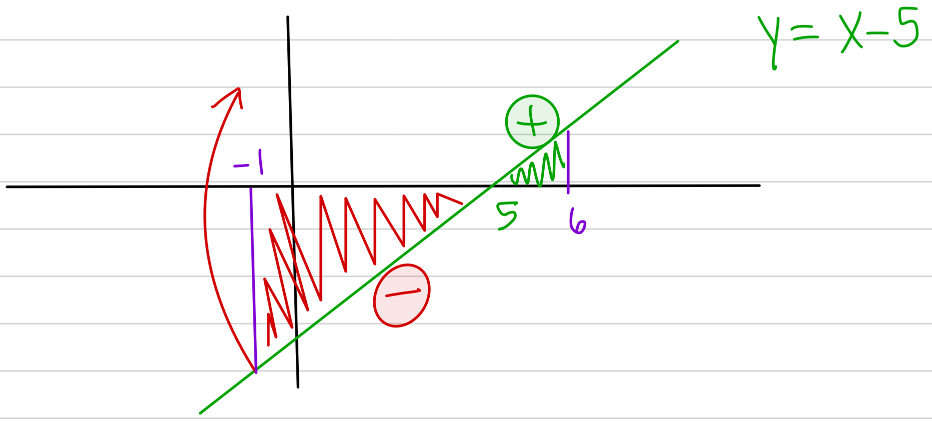
recall $\tan\frac{\pi}{3} = \sqrt{3}$

$-\frac{1}{2\cdot 3}+C=-5$

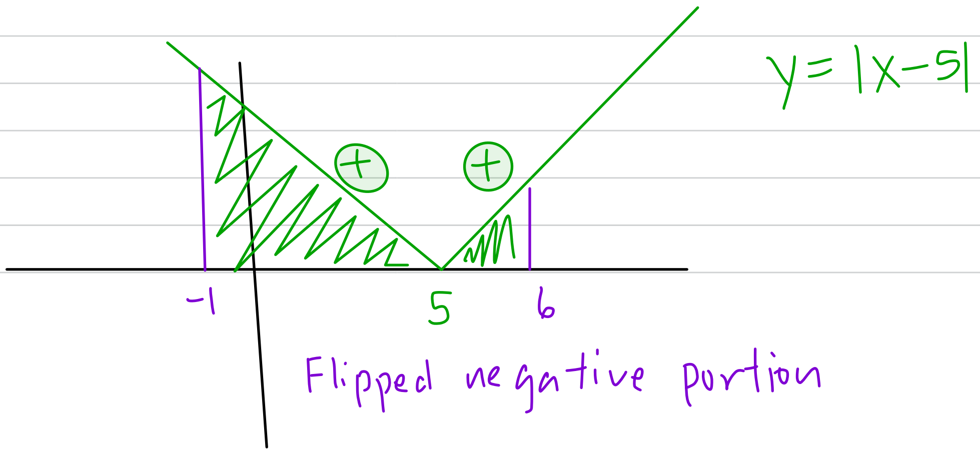
$-\frac{1}{6}+C=-5 \;\xrightarrow{+\frac{1}{6}}\; \Rightarrow C=\frac{-29}{6}$

Finally, $\boxed{f(x) = -\frac{1}{2\tan^2 x} - \frac{29}{6}}$

5(a) $\int_{-1}^{6} x-5\,dx$



5(b) $\int_{-1}^{6} |x-5|\,dx =$

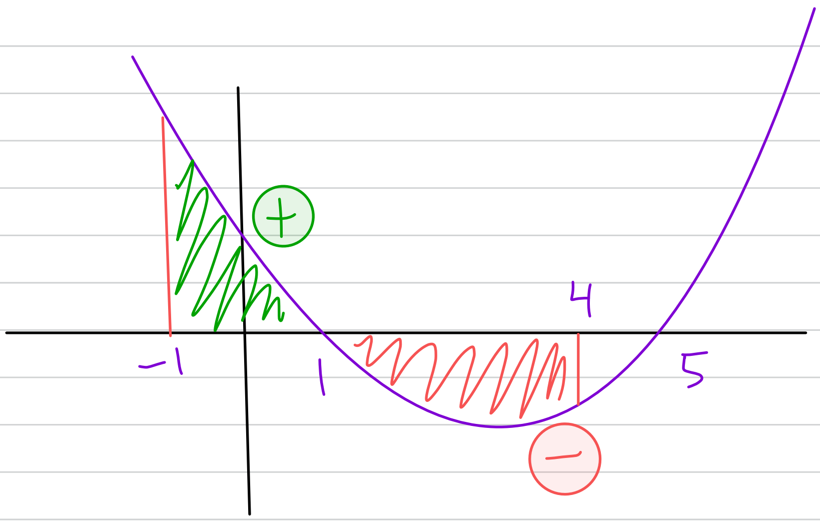


5(c) $\int_{-1}^{4} x^2-6x+5\,dx$

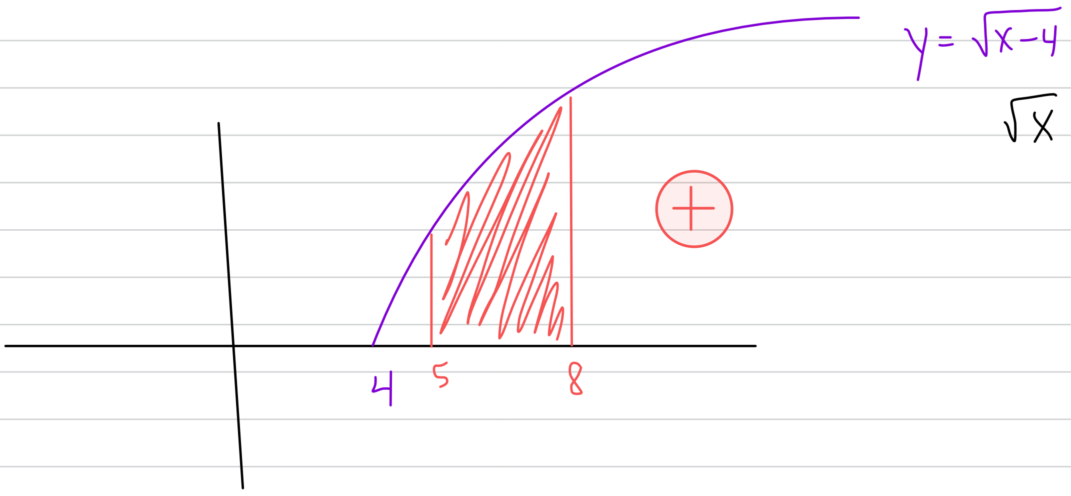
Factor $x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=5$ $x=1$ zeroes



5(d) $\int_5^8 \sqrt{x-4}\,dx$

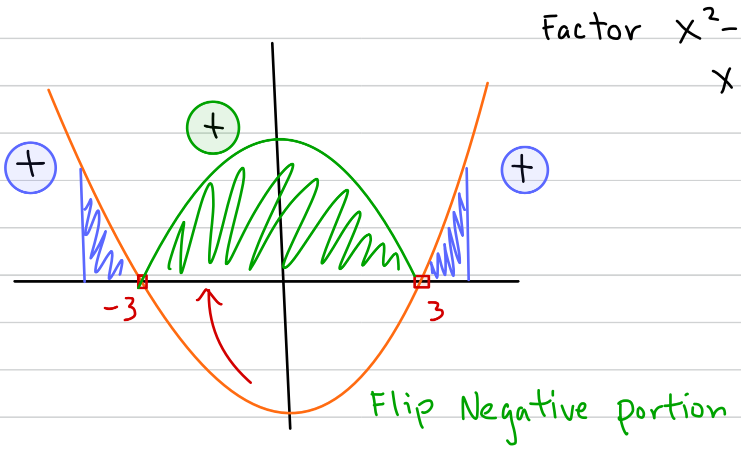


$\sqrt{x}$ shifted right 4 units

5(e) $\int_{-4}^{4} |x^2-9|\,dx$

Factor $x^2-9=0$

$x^2=9 \Rightarrow x=\pm 3$



Bonus Optional

1. $$\int \frac{\cos^3 x}{\sqrt{1-\sin x}}\,dx = \int \frac{\overset{1-\sin^2 x}{\cos^2 x}\cdot \overset{-du}{\cos x\,dx}}{\underset{u}{\sqrt{1-\sin x}}} = -\int \frac{1-\sin^2 x}{\sqrt{u}}\,du = -\int \frac{1-\overset{1-2u+u^2}{(1-u)^2}}{\sqrt{u}}\,du$$

$\sin^2x+\cos^2x=1$ identity

$1-2u+u^2$ FOIL Algebra

$$\boxed{\begin{aligned} u&=1-\sin x \\ du&=-\cos x\,dx \\ -du&=\cos x\,dx \end{aligned}}$$

$\Rightarrow \sin x = 1-u$

$$= -\int \frac{\cancel{1}-\cancel{1}+2u-u^2}{\sqrt{u}}\,du = -\int \frac{2u-u^2}{\sqrt{u}}\,du$$ split

$$= -\int \frac{2u}{\sqrt{u}}-\frac{u^2}{\sqrt{u}}\,du = -\int 2u^{1/2}-u^{3/2}\,du$$

$$= -\left(\frac{2u^{3/2}}{3/2}-\frac{u^{5/2}}{5/2}\right)+C = \boxed{-\left(\frac{4}{3}(1-\sin x)^{3/2}-\frac{2}{5}(1-\sin x)^{5/2}\right)+C}$$

Bonus optional

$$\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\sqrt{4}+\dots+\sqrt{n-1}+\sqrt{n}}{n\sqrt{n}} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\sqrt{n}}\cdot\sqrt{i}$$

reverse

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{i}{n}} = \lim_{n\to\infty}\sum_{i=1}^{n}\underbrace{\sqrt{\frac{i}{n}}}_{f(x_i)}\cdot\underbrace{\frac{1}{n}}_{\Delta x}$$

here $\Delta x = \underbrace{\frac{1}{n}}_{\frac{b-a}{n}}$ and $x_i = \frac{i}{n} = \underbrace{0+\frac{i}{n}}_{a+i\Delta x}$ if $a=0$ and $b-a=1$ then $b=1$

and $f(x) = \sqrt{x}$

$$= \int_0^1 \overset{x^{1/2}}{\sqrt{x}}\,dx = \frac{x^{3/2}}{3/2}\Bigg|_0^1 = \frac{2}{3}x^{3/2}\Bigg|_0^1 = \frac{2}{3}\cdot\cancelto{1}{1^{3/2}} - \cancelto{0}{\frac{2}{3}\cdot 0} = \boxed{\frac{2}{3}}$$