Math 106 Exam 2
March 28, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Compute and show that $\int_{-1}^{2} 2 - 3x - x^2 \, dx = \boxed{-\frac{3}{2}}$ using **ONLY** the **Limit Definition of the Definite Integral**.

2. [16 Points] Evaluate each of the following Indefinite Integrals. Simplify.

(a) $\int \frac{x^6}{\sqrt{3x^7 + 5}} \, dx$

(b) $\int \frac{x}{(x+5)^4} \, dx$

3. [36 Points] Evaluate each of the following Definite Integrals. Simplify.

(a) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^5 x} \, dx = \boxed{\frac{32}{9}}$

(b) Show that $\int_{2}^{6} \frac{1}{x^2} \cdot \cos\left(\frac{\pi}{x}\right) \, dx = \boxed{\frac{1}{2\pi}}$

(c) Show that $\int_{1}^{4} \frac{\frac{1}{x} + 1}{\sqrt{x}} \, dx = \boxed{3}$

(d) Show that $\int_{4}^{9} \frac{3}{\sqrt{x}\,(7 + \sqrt{x}\,)^2} \, dx = \boxed{\frac{1}{15}}$

4. [8 Points] Consider an object travelling with velocity given by $v(t) = t^2 - 4$ feet per second.

Compute the **Total Distance** covered by this object as time ranges from $t = 0$ to $t = 5$. The answer should be positive.

5. [10 Points] Compute $f(x)$ where $f'(x) = \tan^3 x \cdot \sec^2 x$ and $f\left(\frac{\pi}{3}\right) = \frac{1}{4}$

6. [10 Points] **Sketch and Shade** the Bounded Area represented by these Definite Integrals.

DO NOT COMPUTE THE INTEGRAL

(a) $\int_{-2}^{5} x^2 - 3x - 4 \; dx$

(b) $\int_{-4}^{4} |x^2 - 9| \; dx$

(c) $\int_{-2}^{5} 16 - x^2 \; dx$

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 $\int \frac{x^9}{\sqrt{2 - x^2}} \; dx$