Math 106 Exam 1
February 20, 2026

- This is a closed-book examination. No books, notes, calculators, cell phones, electronic communication devices of any sort, webpages, or other aids are permitted.

- Simplify numerical answers such as $\sin\left(\dfrac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [12 Points] **Differentiate** each of the following functions. Do **NOT** simplify your answers.

(a) $f(x) = \tan^6\left(\dfrac{4}{x^2}\right) \stackrel{\text{prep}}{=} \left(\tan\left(\dfrac{4}{x^2}\right)\right)^6$

(b) $f(x) = \sqrt{\cos\sqrt{\sin\sqrt{x}}}$

2. [10 Points] **Derivatives**

Consider $f(x) = \dfrac{6}{\cos^7 x} \stackrel{\text{prep}}{=} \dfrac{6}{(\cos x)^7} \stackrel{\text{prep}}{=} 6\,(\cos x)^{-7}$

(a) Compute the Derivative $f'(x)$ using the Chain Rule.

(b) Start again. Compute the Derivative $f'(x)$ using the Quotient Rule this time.

Show/Check that the answers are the same

3. [25 Points] **Derivatives**

(a) Let $f(x) = \sqrt{3}\cdot\cos(5x) + \cos(3x) + \sin(3x) + \sin(2x)$ Show that $f'\left(\dfrac{\pi}{3}\right) = \boxed{\dfrac{7}{2}}$

(b) Let $f(x) = \sqrt{\sin(3x)} + \sin(8x)$.   Show that $f'\left(\dfrac{\pi}{6}\right) = \boxed{-4}$

(c) Let $H(x) = \tan(6x) + \cos^2(2x) + 2\cos(3x) \stackrel{\text{prep}}{=} \tan(6x) + (\cos(2x))^2 + 2\cos(3x)$

Show that $H'\left(\dfrac{\pi}{6}\right) = \boxed{-\sqrt{3}}$

4. [20 Points] Compute the following **Most General Antiderivatives.**

(a) $\displaystyle \int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - \frac{1}{x^{\frac{3}{7}}} + \frac{1}{7} + \frac{1}{3x^{\frac{7}{3}}} - \frac{1}{7x^3} - \frac{3}{x^7} \; dx$

(b) $\displaystyle \int \left(x^3 + \frac{1}{x^3} \right) \left(x - \frac{1}{x} \right) \; dx$

(c) $\displaystyle \int \frac{x^7 - 4x^3 - \dfrac{8}{x} + \sqrt{x} + 5 - x^3 \cdot \sec^2 x}{x^3} \; dx$

5. [15 Points] A small rocket is launched straight upwards from the ground at 100 miles per minute. There is a photographer on the ground 5 miles from the launch pad. Consider the Angle θ at the rocket, as shown in the diagram. How fast is this Angle θ changing when the (diagonal) distance between the rocket and the Photographer is 10 miles?

<mark>LABEL all steps</mark>

• Diagram

Rocket
θ

5

Photographer
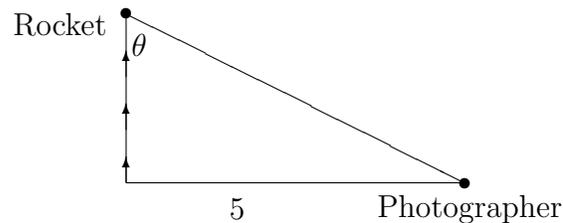
6. [12 Points] For the following, use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

A ball is thrown *upwards* from the edge of the top of a building that is 80 feet tall with an initial velocity of 64 feet per second. Answer the following questions.

• **What** is the Maximum Height reached by the ball?

• **What** is the Velocity of the ball at impact with the ground?

HINT: $-16t^2 + 64t + 80 = -16(t^2 - 4t - 5)$