Exam 1 Spring 2026 Answer Key

1(a) $f(x) = \tan^6\left(\frac{4}{x^2}\right) \overset{\text{prep}}{=} \left(\tan\left(\frac{4}{x^2}\right)\right)^6$

$\frac{d}{dx}$ $4x^{-2} \hookrightarrow -8x^{-3}$

$$f'(x) = \boxed{6\left(\tan\left(\frac{4}{x^2}\right)\right)^5 \cdot \sec^2\left(\frac{4}{x^2}\right) \cdot \left(-\frac{8}{x^3}\right)}$$

1(b) $f(x) = \sqrt{\cos\sqrt{\sin\sqrt{x}}}$

$$f'(x) = \boxed{\frac{1}{2\sqrt{\cos\sqrt{\sin\sqrt{x}}}} \cdot \left(-\sin\sqrt{\sin\sqrt{x}}\right) \cdot \frac{1}{2\sqrt{\sin\sqrt{x}}} \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}}$$

2(a) $f(x) = \frac{6}{\cos^7 x} \overset{\text{prep}}{=} \frac{6}{(\cos x)^7} = 6(\cos x)^{-7}$

Chain Rule

$$f'(x) = \cancel{-}42(\cos x)^{-8} \cdot (\cancel{-}\sin x) = \boxed{\frac{42\sin x}{\cos^8 x}}$$

2(b) Quotient Rule

$$f'(x) = \frac{\cancel{\cos^7 x \cdot \overset{0}{0}} - 6 \cdot 7(\cos x)^6 \cdot (\cancel{-}\sin x)}{(\cos^7 x)^2} \qquad (\cos x)^{14}$$

$$= \boxed{\frac{42\sin x}{\cos^8 x}}$$

MATCH!

3(a) $f(x) = \sqrt{3}\cos(5x) + \cos(3x) + \sin(3x) + \sin(2x)$

$f'(x) = \sqrt{3} \cdot (-\sin(5x)) \cdot 5 - \sin(3x) \cdot 3 + \cos(3x) \cdot 3 + \cos(2x) \cdot 2$

$= -5\sqrt{3}\sin(5x) - 3\sin(3x) + 3\cos(3x) + 2\cos(2x)$

$$f'\left(\frac{\pi}{3}\right) = -5\sqrt{3}\,\overset{-\frac{\sqrt{3}}{2}}{\sin\left(\frac{5\pi}{3}\right)} - 3\,\overset{0}{\sin\left(\frac{\cancel{3}\pi}{\cancel{3}}\right)} + 3\,\overset{-1}{\cos\left(\frac{\cancel{3}\pi}{\cancel{3}}\right)} + 2\,\overset{-1/2}{\cos\left(\frac{2\pi}{3}\right)}$$

$\pi \qquad \pi$

$$= +\frac{5 \cdot 3}{2} - 0 - 3 - 1 = \frac{15}{2} - 4 = \frac{15}{2} - \frac{8}{2} = \boxed{\frac{7}{2}}$$

MATCH!

3(b) $f(x) = \sqrt{\sin(3x)} + \sin(8x)$

$$f'(x) = \frac{1}{2\sqrt{\sin(3x)}} \cdot \cos(3x) \cdot 3 + 8\cos(8x)$$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{2\sqrt{\sin\left(\frac{3\pi}{6}\right)}} \cdot \cos\left(\frac{3\pi}{6}\right) \cdot 3 + 8\cos\left(\frac{8\pi}{6}\right)$$

$\frac{\pi}{2}$ $\quad$ 1 $\quad$ $\frac{\pi}{2}$ $\quad$ 0 $\quad$ 0 $\quad$ $\frac{4\pi}{3}$ $\quad$ $-\frac{1}{2}$

$$= 0 + 8\left(-\frac{1}{2}\right) = \boxed{-4}$$

MATCH!

☆ $(\sec \pi)^2 = \left(\frac{1}{\cos \pi}\right)^2$ $\quad$ -1 $\quad$ 1

3(c) $f(x) = \tan(6x) + (\cos(2x))^2 + 2\cos(3x)$

$$f'(x) = \sec^2(6x) \cdot 6 + 2\cos(2x)(-\sin(2x)) \cdot 2 + 2(-\sin(3x)) \cdot 3$$

$$= 6\sec^2(6x) - 4\cos(2x)\sin(2x) - 6\sin(3x)$$

$$f'\left(\frac{\pi}{6}\right) = 6\sec^2\left(\cancel{6} \cdot \frac{\pi}{\cancel{6}}\right) - 4\cos\left(\frac{\cancel{2}\pi}{\cancel{6}}\right)\sin\left(\frac{\cancel{2}\pi}{\cancel{6}}\right) - 6\sin\left(\frac{\cancel{3}\pi}{\cancel{6}}\right)$$

☆ π $\quad$ 1 $\quad$ $\frac{\pi}{3}$ $\quad$ $\frac{1}{2}$ $\quad$ $\frac{\pi}{3}$ $\quad$ $\frac{\sqrt{3}}{2}$ $\quad$ $\frac{\pi}{2}$ $\quad$ 1

$$= \cancel{6} - \cancel{4}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \cancel{6} = \boxed{-\sqrt{3}}$$

MATCH!

4(a) $\int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - \frac{1}{x^{\frac{3}{7}}} + \frac{1}{7} + \frac{1}{3x^{7/3}} - \frac{1}{7x^3} - \frac{3}{x^7}\,dx$

prep

$$= \int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - x^{-\frac{3}{7}} + \frac{1}{7} + \frac{1}{3}x^{-\frac{7}{3}} - \frac{1}{7}x^{-3} - 3x^{-7}\,dx$$

Don't Move Constants

$$= \frac{3}{7} \cdot \frac{x^2}{2} + \frac{x^{\frac{10}{7}}}{10/7} + \frac{7}{3} \cdot \frac{x^4}{4} - \frac{x^{\frac{4}{7}}}{4/7} + \frac{1}{7}x + \frac{1}{3}\left(\frac{x^{-\frac{4}{3}}}{-4/3}\right) - \frac{1}{7}\left(\frac{x^{-2}}{-2}\right) - \frac{3x^{-6}}{-6} + C$$

$$= \boxed{\frac{3x^2}{14} + \frac{7}{10}x^{\frac{10}{7}} + \frac{7}{12}x^4 - \frac{7}{4}x^{\frac{4}{7}} + \frac{1}{7}x - \frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{3}}x^{-\frac{4}{3}} + \frac{1}{14}x^{-2} + \frac{1}{2}x^{-6} + C}$$

$-\frac{1}{4}$

4(b) $\int \left(x^3 + \frac{1}{x^3}\right)\left(x - \frac{1}{x}\right) dx \overset{\text{FOIL}}{=} \int x^4 - x^2 + x^{-2} - x^{-4}\,dx$

x^{-3} $\quad$ x^{-1}

$$= \frac{x^5}{5} - \frac{x^3}{3} + \frac{x^{-1}}{-1} \cancel{-} \frac{x^{-3}}{\cancel{-}3} + C$$

$$= \boxed{\frac{x^5}{5} - \frac{x^3}{3} - \frac{1}{x} + \frac{1}{3x^3} + C}$$

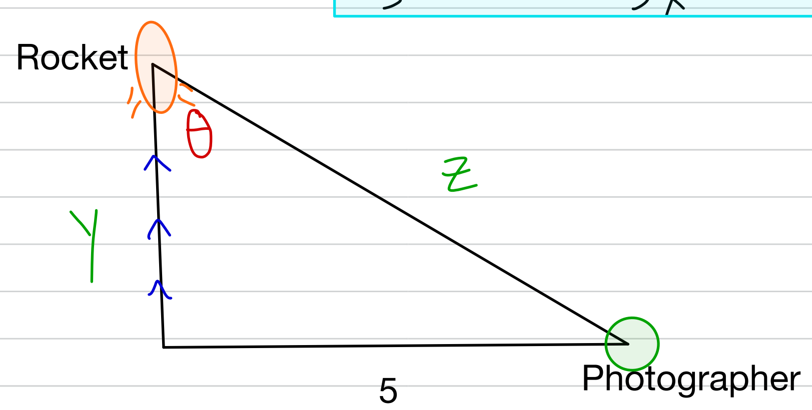
4(c) $\int \frac{x^7-4x^3-\frac{8}{x}+\sqrt{x}+5-x^3\sec^2 x}{x^3}\,dx$ split $= \int \frac{x^7}{x^3}-\cancel{\frac{4x^3}{x^3}}-\frac{8x^{-1}}{x^3}+\frac{x^{1/2}}{x^3}+\frac{5}{x^3}-\cancel{\frac{x^3\sec^2 x}{x^3}}\,dx$

$$= \int x^4-4-8x^{-4}+x^{-\frac{5}{2}}+5x^{-3}-\sec^2 x\,dx$$

$$= \frac{x^5}{5}-4x-\frac{8x^{-3}}{-3}+\frac{x^{-3/2}}{-3/2}+\frac{5x^{-2}}{-2}-\tan x+C$$

$$= \boxed{\frac{x^5}{5}-4x+\frac{8}{3x^3}-\frac{2}{3x^{3/2}}-\frac{5}{2x^2}-\tan x+C}$$

5. Diagram → given



Variables

Let y = Distance the Rocket has travelled vertically

z = Distance between Rocket and Photographer

θ = Angle at the Rocket corner

Given $\frac{dy}{dt} = 100$ miles/min

Find $\frac{d\theta}{dt} = ?$ when $z = 10$

Equation $\tan\theta = \frac{5}{y}$

Differentiate

$$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(5y^{-1}\right)$$

$(\sec\theta)^2 \rightarrow$ $\sec^2\theta \cdot \frac{d\theta}{dt} = -5y^{-2}\cdot\frac{dy}{dt}$

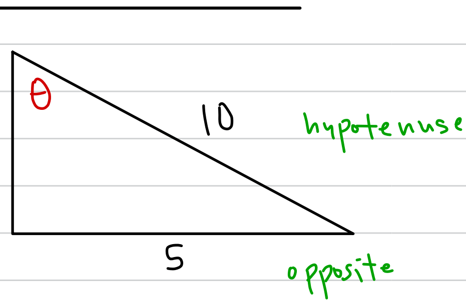
Extra Solvable Information

Adjacent

$\Rightarrow \sqrt{(10)^2-5^2}$

$= \sqrt{100-25}$

$\sqrt{75}$



$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{H}{A} = \frac{10}{\sqrt{75}}$

Substitute

$$\left(\frac{10}{\sqrt{75}}\right)^2 \cdot \frac{d\theta}{dt} = -\frac{5}{(\sqrt{75})^2}\cdot(100)$$

?

$\frac{100}{(\sqrt{75})^2}$ will Cancel

Solve

$$\frac{d\theta}{dt} = -\frac{5}{\cancel{75}}\left(\cancel{100}\right)\cdot\frac{\cancel{75}}{\cancel{100}}$$

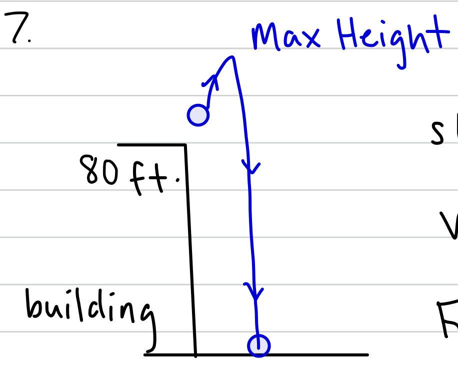
$$= -5 \text{ Radians/min}$$

Minus makes sense since Angle shrinking as rocket flies

Answer

$\boxed{\text{The Angle at the Rocket corner is decreasing 5 Radians every minute at that Moment}}$

7.



$s(0) = 80$ ft.

$v(0) = 64$ ft/sec

Find $t_{max} = ?$

Find Max Height $\hookrightarrow s(t_{max}) = ?$

Find $t_{impact} = ?$

Find $v(t_{impact}) = ?$

$a(t) = -32$

$v(t) = -32t + \cancel{v_0}^{64}$

$= -32t + 64$

$s(t) = -16t^2 + \cancel{v_0}^{64} t + \cancel{s_0}^{80}$

$= -16t^2 + 64t + 80$

(a) Max Height is when $v(t) = 0$

$v(t) = -32t + 64 \overset{set}{=} 0$ Solve $32t = 64 \Rightarrow t_{max} = \frac{64}{32} = 2$ sec

Max Height occurs at $t = 2$ seconds

(b) Max Height occurs when $t_{max} = 2$ seconds

$$\Rightarrow s(t_{max}) = s(2) = -16\overset{4}{(2)^2} + 64(2) + 80$$

Fractions will cancel

$$= \underbrace{-64 + 128}_{64} + 8 = \boxed{72 \text{ feet}}$$

$\boxed{\text{The Maximum Height is 72 feet}}$

(c) Strikes ground when $s(t) = 0$

$$s(t) = -16t^2 + 64t + 80 = -16(t^2 - 4t - 5)$$

$$= -16(t-5)(t+1) \overset{set}{=} 0$$

$t - 5 = 0$ OR $t + 1 = 0$

$t_{impact} = 5$ Sec. $\cancel{t = -1 \text{ sec}}$

$\boxed{\text{Hits ground after } t_{impact} = 5 \text{ seconds}}$

(d) Velocity at impact $v(t_{impact}) = v(5) = \underbrace{-32 \cdot 5}_{-160} + 64 = -96$ ft/sec

negative makes sense. DOWN @ impact.

$\boxed{\text{Impact Velocity is } -96 \text{ feet per second}}$