Math 106 Exam 1
February 21, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, electronic communication devices of any sort, webpages, or other aids are permitted.

• Simplify numerical answers such as $\sin\left(\frac{\pi}{6}\right)$ and $4^{\frac{3}{2}}$.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [21 Points] **Differentiate** each of the following functions. Do **NOT** simplify.

(a) $f(x) = \tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{6}{x}\right)$

(b) $f(x) = \cos(\sin(\sec x))$

(c) $f(x) = \sin^6\left(\frac{5}{x^4}\right) \overset{\text{prep}}{=} \left(\sin\left(\frac{5}{x^4}\right)\right)^6$

(d) $f(x) = 5\sin^2 x + 5\cos^2 x$

(e) $f(x) = \dfrac{6}{\sqrt{\cos\sqrt{x}}} \overset{\text{prep}}{=} 6\left(\cos\sqrt{x}\right)^{-\frac{1}{2}}$

2. [20 Points] **Derivatives**

(a) Let $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$ Show that $f'\left(\frac{\pi}{6}\right) = \boxed{\frac{3}{2}}$

(b) Let $H(x) = \cos^2(2x) + \sin(6x) + 2\sin x$ Show that $H'\left(\frac{\pi}{6}\right) = \boxed{-6}$

3. [24 Points] Compute the following **Most General Antiderivatives.**

(a) $\displaystyle\int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - \frac{1}{x^{\frac{3}{7}}} + \frac{1}{7} + \frac{1}{3x^{\frac{7}{3}}} - \frac{1}{7x^3} - \frac{3}{x^7}\,dx$

(b) $\displaystyle\int \left(x^3 + \frac{1}{x^3}\right)\left(x - \frac{1}{x}\right)\,dx$

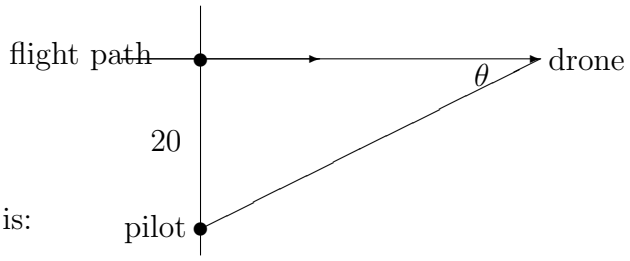
(c) $\displaystyle\int \frac{x^7 - 4x^3 - \frac{8}{x} + \sqrt{x} + 5 - x^3 \cdot \sec^2 x}{x^3}\,dx$

4. [6 Points] **PROVE** that $\dfrac{d}{dx}\tan x = \sec^2 x$

5. [15 Points] A drone is flying exactly 20 feet above its pilot on the ground. Suppose that the drone is flying horizontally at 10 feet per second. Consider the Angle θ as shown in the diagram. How fast is this Angle θ changing when the (diagonal) distance between the pilot and the drone is 40 feet?

• Diagram

The picture at arbitrary time t is:



6. [8 Points] Consider a function G such that $G'(x) = \dfrac{18}{x^2} - \dfrac{1}{\sqrt{x}} + 3$ and $G(9) = 7$

Compute $G(x)$.

7. [12 Points] For the following, use $a(t) = -32$ feet per second squared as acceleration due to gravity on the falling body.

A ball is thrown *upwards* from the edge of the top of a building that is 96 feet tall with an initial velocity of 80 feet per second. Answer the following questions.

• **When** does the ball reach its Maximum Height?

• **What** is the Maximum Height reached by the ball?

• **When** does the ball strike the ground?

• **What** is the Velocity of the ball at impact with the ground?

HINT: $-16t^2 + 80t + 96 = -16(t^2 - 5t - 6)$