Exam 1 Spring 2025 Answer Key

1(a) $f(x) = \underbrace{\tan\left(\frac{\pi}{6}\right)}_{\text{constant}} + \tan\left(\frac{6}{x}\right) \overset{\text{prep}}{=} \tan\left(\frac{\pi}{6}\right) + \tan\left(6x^{-1}\right)$

$f'(x) = \boxed{0 + \sec^2\left(\frac{6}{x}\right) \cdot \left(-6x^{-2}\right)}$

Do Not Need to Simplify

1(b) $f(x) = \cos\left(\sin\left(\sec x\right)\right)$

$f'(x) = -\sin\left(\sin\left(\sec x\right)\right) \cdot \cos\left(\sec x\right) \cdot \sec x \tan x$

1(c) $f(x) = \sin^6\left(\frac{5}{x^4}\right) \overset{\text{prep}}{=} \left(\sin\left(\frac{5}{x^4}\right)\right)^6$

prep $5x^{-4} \xrightarrow{\frac{d}{dx}} -20x^{-5}$

$f'(x) = \boxed{6\left(\sin\left(\frac{5}{x^4}\right)\right)^5 \cdot \cos\left(\frac{5}{x^4}\right) \cdot \left(-20x^{-6}\right)}$

1(d) $f(x) = 5\sin^2 x + 5\cos^2 x \overset{\text{prep}}{=} 5\left(\cancel{\sin^2 x + \cos^2 x}^{\,1}\right) = 5$ constant

$f'(x) = \boxed{0}$

OR//

$f(x) = 5\left(\sin x\right)^2 + 5\left(\cos x\right)^2$

$f'(x) = 10\left(\sin x\right)^1 \cdot \cos x + 10\left(\cos x\right)^1\left(-\sin x\right)$

$= \cancel{10 \sin x \cos x} - \cancel{10 \sin x \cdot \cos x} = \boxed{0}$

1(e) $f(x) = \frac{6}{\sqrt{\cos\sqrt{x}}} \overset{\text{prep}}{=} 6\left(\cos\sqrt{x}\right)^{-1/2}$

$f'(x) = \boxed{-3\left(\cos\sqrt{x}\right)^{-\frac{3}{2}} \cdot \left(-\sin\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}}$

2(a) $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$

$f'(x) = -7\sin(7x) - 6\sin(6x) + 3\cos(3x) + 4\cos(4x)$

$f'\left(\frac{\pi}{6}\right) = -7\sin\left(\frac{7\pi}{6}\right) - 6\sin\left(\cancel{6}\cdot\frac{\pi}{\cancel{6}}\right) + 3\cos\left(\cancel{3}\frac{\pi}{\cancel{6}_{2}}\right) + 4\cos\left(\cancel{4}^{2}\cdot\frac{\pi}{\cancel{6}_{3}}\right)$

$= -7\underbrace{\cancel{\sin\left(\frac{7\pi}{6}\right)}^{-\frac{1}{2}}}_{\text{See 4. above}} - 6\cancel{\sin(\pi)}^{0} + 3\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} + 4\underbrace{\cancel{\cos\left(\frac{2\pi}{3}\right)}^{-\frac{1}{2}}}_{\text{See 5. above}}$

$= --\frac{7}{2} + 0 + 0 - 2 = \frac{7}{2} - 2 = \frac{7}{2} - \frac{4}{2} = \frac{3}{2}$ Match

2(b) $H(x) = \cos^2(2x) + \sin(6x) + 2\sin x$

$= (\cos(2x))^2 + \sin(6x) + 2\sin x$

$H'(x) = 2\cos(2x)\cdot(-\sin(2x))\cdot 2 + 6\cos(6x) + 2\cos x$

$H'\left(\frac{\pi}{6}\right) = -4\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right)\cdot\sin\left(\cancel{2}\cdot\frac{\pi}{\cancel{6}_3}\right) + 6\cos\left(\cancel{6}\cdot\frac{\pi}{\cancel{6}}\right) + 2\cos\left(\frac{\pi}{6}\right)$

$= -4\underbrace{\cos\left(\frac{\pi}{3}\right)}_{\frac{1}{2}}\cdot\underbrace{\sin\left(\frac{\pi}{3}\right)}_{\frac{\sqrt{3}}{2}} + 6\underbrace{\cos\pi}_{-1} + 2\underbrace{\cos\left(\frac{\pi}{6}\right)}_{\frac{\sqrt{3}}{2}}$

$= -\cancel{4}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{\cancel{2}}\right) + 6(-1) + \cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}$

$= -\cancel{\sqrt{3}} - 6 + \cancel{\sqrt{3}} = -6$ Match!

3(a) $\int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - \frac{1}{x^{\frac{3}{7}}} + \frac{1}{7} + \frac{1}{3x^{7/3}} - \frac{1}{7x^3} - \frac{3}{x^7}\,dx$

prep
$= \int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - x^{-\frac{3}{7}} + \frac{1}{7} + \frac{1}{3}x^{-\frac{7}{3}} - \frac{1}{7}x^{-3} - 3x^{-7}\,dx$

Don't Move Constants

$= \frac{3}{7}\cdot\frac{x^2}{2} + \frac{x^{\frac{10}{7}}}{10/7} + \frac{7}{3}\cdot\frac{x^4}{4} - \frac{x^{\frac{4}{7}}}{4/7} + \frac{1}{7}x + \frac{1}{3}\left(\frac{x^{-\frac{4}{3}}}{-4/3}\right) - \frac{1}{7}\left(\frac{x^{-2}}{-2}\right) - \frac{3x^{-6}}{-6} + C$

$$\boxed{= \frac{3x^2}{14} + \frac{7}{10}x^{\frac{10}{7}} + \frac{7}{12}x^4 - \frac{7}{4}x^{\frac{4}{7}} + \frac{1}{7}x - \frac{\cancel{3}}{4}\cdot\frac{1}{\cancel{3}}x^{-\frac{4}{3}} + \frac{1}{14}x^{-2} + \frac{1}{2}x^{-6} + C}$$

$-\frac{1}{4}$

3(b) $\int\left(x^3 + \frac{1}{x^3}\right)\left(x - \frac{1}{x}\right)dx \overset{\text{FOIL}}{=} \int x^4 + x^2 + x^{-2} - x^{-4}\,dx$

x^{-3} $\quad x^{-1}$

$= \frac{x^5}{5} + \frac{x^3}{3} + \frac{x^{-1}}{-1} \cancel{-} \frac{x^{-3}}{\cancel{-}3} + C$

$$\boxed{= \frac{x^5}{5} + \frac{x^3}{3} - \frac{1}{x} + \frac{1}{x^3} + C}$$

3(c) $\int \frac{x^7 - 4x^3 - \frac{8}{x} + \sqrt{x} + 5 - x^3\sec^2 x}{x^3}\,dx \overset{\text{split}}{=} \int \frac{x^7}{x^3} - \frac{4\cancel{x^3}}{\cancel{x^3}} - \frac{8x^{-1}}{x^3} + \frac{x^{1/2}}{x^3} + \frac{5}{x^3} - \frac{\cancel{x^3}\sec^2 x}{\cancel{x^3}}\,dx$

$= \int x^4 - 4 - 8x^{-4} + x^{-\frac{5}{2}} + 5x^{-3} - \sec^2 x\,dx$

$= \frac{x^5}{5} - 4x \cancel{-} \frac{8x^{-3}}{\cancel{-}3} + \frac{x^{-3/2}}{-3/2} + \frac{5x^{-2}}{-2} - \tan x + C$

$$\boxed{= \frac{x^5}{5} - 4x + \frac{8}{3x^3} - \frac{2}{3x^{3/2}} - \frac{5}{2x^2} - \tan x + C}$$

4. Prove $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot (\cos x) \cancel{-} (\sin x)(\cancel{-}\sin x)}{(\cos x)^2}$$

$+$

Quotient Rule

Trig Identity

$$= \frac{\cancel{\cos^2 x + \sin^2 x}^{\,1}}{\cos^2 x} = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2 = \sec^2 x \checkmark$$

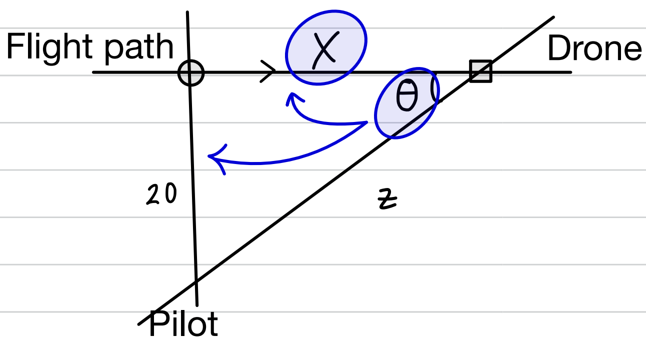
Match

OR

split

$$= \cancel{\frac{\cos^2 x}{\cos^2 x}} + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \sec^2 x \checkmark$$

4. Diagram → given



Variables

Let x = Distance the drone has travelled horizontally

z = Distance between drone and pilot

θ = Angle at the drone corner

Given $\frac{dx}{dt} = 10$ feet/sec

Find $\frac{d\theta}{dt} = ?$ when $z = 40$

Equation

$$\tan\theta = \frac{20}{x}$$

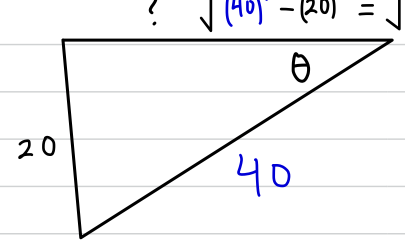
Differentiate

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(20x^{-1})$$

$$(\sec\theta)^2 \rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = -20x^{-2} \cdot \frac{dx}{dt}$$

Extra Solvable Information

$? \quad \sqrt{(40)^2 - (20)^2} = \sqrt{1600 - 400} = \sqrt{1200} \overset{\text{or}}{=} 20\sqrt{3}$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{H}{A} = \frac{40}{\sqrt{1200}}$$

Substitute

$$\left(\frac{40}{\sqrt{1200}}\right)^2 \cdot \frac{d\theta}{dt} = -\frac{20}{(\sqrt{1200})^2} \cdot (10)$$

?

$$\frac{(40)^2}{(\sqrt{1200})^2}$$

Solve

$$\frac{d\theta}{dt} = -\frac{20}{\cancel{1200}}(10)\cdot\frac{\cancel{1200}}{1600}$$

Minus makes sense since Angle shrinking as drone flies

$$= \frac{-2\cancel{00}}{16\cancel{00}} = -\frac{1}{8} \text{ Radians/sec.}$$

Answer

The Angle at the drone corner is decreasing $\frac{1}{8}$ Radians every second at that Moment

5. $G'(x) = \frac{18}{x} - \frac{1}{\sqrt{x}} + 3$ and $G(9) = 7$

use Initial Condition to find +C

Antidifferentiate $G(x) = \int G'(x)\,dx = \int \frac{18}{x^2} - \frac{1}{\sqrt{x}} + 3 \, dx$

$$= \int 18x^{-2} - x^{-\frac{1}{2}} + 3\,dx$$

$$= 18\cdot\frac{x^{-1}}{-1} - \frac{x^{1/2}}{1/2} + 3x + C$$

$$= -\frac{18}{x} - 2\sqrt{x} + 3x + C$$

Test Initial Condition

$$G(9) = -\frac{\cancel{18}^{\,-2}}{\cancel{9}} - 2\sqrt{\cancel{9}}^{\,3} + \overset{3\cdot 9}{27} + C \overset{\text{set}}{=} 7$$

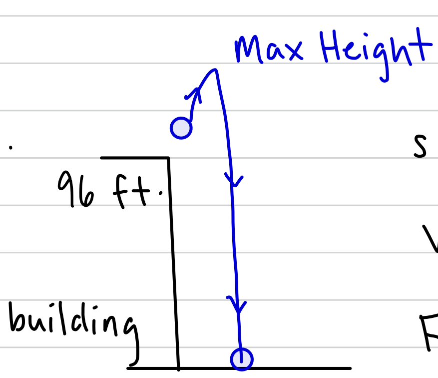
$$-2 - 3 + 27 + C = 7$$

$$-5 + 27$$

$$22 + C = 7 \Rightarrow C = -15$$

Finally, $\boxed{G(x) = \frac{-18}{x} - 2\sqrt{x} + 3x - 15}$

6.



$s(0) = 96$ ft.

$v(0) = 80$ ft/sec

Find $t_{max} = ?$

Find Max Height $\hookrightarrow s(t_{max}) = ?$

Find $t_{impact} = ?$

Find $v(t_{impact}) = ?$

$a(t) = -32$

$v(t) = -32t + \cancel{v_0}^{\,80}$

$= -32t + 80$

$s(t) = -16t^2 + \cancel{v_0}^{\,80}t + \cancel{s_0}^{\,96}$

$= -16t^2 + 80t + 96$

(a) Max Height is when $v(t)=0$

$v(t) = -32t + 80 \overset{\text{set}}{=} 0$ ↳ Solve $32t = 80 \Rightarrow t_{max} = \frac{80}{32} = \frac{10}{4} = \frac{5}{2}$ sec

2.5

Max Height occurs at t = 2 seconds

(b) Max Height occurs when $t_{max} = \frac{5}{2}$ seconds

$$\Rightarrow s(t_{max}) = s\left(\frac{5}{2}\right) = -16\left(\frac{5}{2}\right)^2 + \overset{40}{\cancel{80}}\left(\frac{5}{\cancel{2}}\right) + 96$$

$$= -\overset{4}{\cancel{16}}\left(\frac{25}{\cancel{4}}\right) + 200 + 96$$

$$= \underbrace{-100 + 200}_{100} + 96 = \boxed{196 \text{ feet}}$$

$\boxed{\text{The Maximum Height is 196 feet}}$

(c) Strikes ground when $s(t) = 0$

$$s(t) = -16t^2 + 80t + 96 = -16(t^2 - 5t - 6)$$

$$= -16(t-6)(t+1) \overset{\text{set}}{=} 0$$

$t - 6 = 0$ OR $t + 1 = 0$

$t = 6$ Sec. $\cancel{t = -1 \text{ sec}}$

$\boxed{\text{Hits ground after } t_{impact} = 6 \text{ seconds}}$

(d) Velocity at impact $v(t_{impact}) = v(6) = \underbrace{-32 \cdot 6}_{-192} + 80 = -112$ ft/sec

negative makes sense. Down @ impact.

$\boxed{\text{Impact Velocity is } -112 \text{ feet per second}}$