Math 106 Final Exam Spring 2025

1(a) $$\frac{d}{dx}\ln\left(\frac{\sqrt{8-x^5}\cdot\tan(e^x)}{(1-e^x)^7\cdot e^{-4x}}\right)$$ First use Log Algebra to simplify

$$=\frac{d}{dx}\ln\left(\sqrt{8-x^5}\cdot\tan(e^x)\right)-\ln\left((1-e^x)^7\cdot e^{-4x}\right)$$

$$=\frac{d}{dx}\ln(8-x^5)^{\frac{1}{2}}+\ln(\tan(e^x))-\left(\ln((1-e^x)^7)+\overset{-4x}{\cancel{\ln(e^{-4x})}}\right)$$

minuses cancel on last term

$$\overset{\text{prep}}{=}\frac{d}{dx}\frac{1}{2}\ln(8-x^5)+\ln(\tan(e^x))-7\ln(1-e^x)\overset{--}{+}4x$$

$$=\boxed{\frac{1}{2}\left(\frac{1}{8-x^5}\right)\cdot(-5x^4)+\frac{1}{\tan(e^x)}\cdot\sec^2(e^x)\cdot e^x-7\left(\frac{1}{1-e^x}\right)\cdot(-e^x)+4}$$

1(b) Let $y=5^x$

$\ln y=\ln(5^x)=x\cdot\ln 5$

$\frac{d}{dx}(\ln y)=\frac{d}{dx}(\ln 5\cdot x)$ $\ln 5=$ constant

$\frac{1}{y}\frac{dy}{dx}=\ln 5$

$\frac{dy}{dx}=\overset{5^x}{\cancel{y}}\cdot\ln 5=\boxed{5^x\ \ln 5}$ ✓

1(c) $f(x)=x^2\cdot e^x$

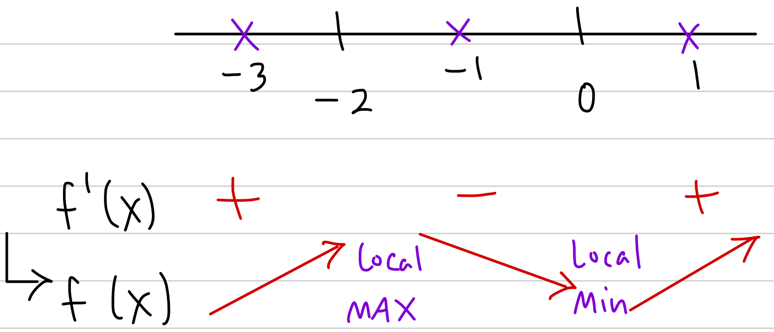
$f'(x)=x^2\cdot e^x+e^x(2x)=e^x(x^2+2x)=\overset{+}{e^x}\cdot x(x+2)\overset{\text{set}}{=}0$

$e^x\neq 0$

$\Rightarrow x=0$ or $x+2=0$

↳ $x=-2$ Critical Numbers

Sign Testing into $f'(x)$



Note e^x Always ⊕

$f'(-3)=\overset{+}{e^{-3}}\overset{-}{(-3)}\overset{-}{(-3+2)}=+$

$f'(-1)=\overset{+}{e^{-1}}\overset{-}{(-1)}\overset{+}{(-1+2)}=-$

$f'(1)=\overset{+}{e^{1}}\overset{+}{(1)}\overset{+}{(1+2)}=+$

☆ $f(x)$ has Local Maximum Value of $f(-2)=4e^{-2}=\frac{4}{e^2}$ occuring when $x=-2$

$f(x)$ has Local Minimum Value of $f(0)=0$ occuring when $x=0$

1(d) $y=[\ln(x+4)]^2$

$$y'=\frac{2(\ln(x+4))}{x+4}\overset{\text{set}}{=}0 \Rightarrow \ln(x+4)=0$$

$$\Rightarrow \cancel{e}^{\ln(x+4)}=e^{0}$$

$$x+4=1$$

$$x=-3 \Rightarrow y=[\ln(-3+4)]^2=[\overset{0}{\cancel{\ln 1}}]^2=0$$

Finally, point $(x,y)=\boxed{(-3,0)}$

2(a) $f(x)=2\sin(5x)+\sin(6x)-5\cos(2x)-\sin(3x)$

$$f'(x)=10\cos(5x)+6\cos(6x)\overset{--}{+}10\sin(2x)-3\cos(3x)$$

$$f'\left(\frac{\pi}{6}\right)=10\,\overset{-\frac{\sqrt{3}}{2}}{\cancel{\cos\left(\frac{5\pi}{6}\right)}}+6\,\overset{-1}{\cancel{\cos\left(6\left(\frac{\pi}{6}\right)\right)}}+10\,\overset{\frac{\sqrt{3}}{2}}{\cancel{\sin\left(\underset{3}{2\left(\frac{\pi}{6}\right)}\right)}}-3\,\overset{0}{\cancel{\cos\left(\underset{2}{3\left(\frac{\pi}{6}\right)}\right)}}$$

$$=\cancel{-10\left(\frac{\sqrt{3}}{2}\right)}-6+\cancel{10\left(\frac{\sqrt{3}}{2}\right)}-0=\boxed{-6}$$ Match!

2(b) $f(x)=\cos^2(3x)+\tan x=(\cos(\ x))^2+\tan x$

$$f'(x)=2\cos(3x)\cdot(-\sin(3x))\cdot 3+\sec^2 x$$

$$\sec\left(\frac{\pi}{6}\right)=\frac{1}{\underset{\frac{\sqrt{3}}{2}}{\cos\left(\frac{\pi}{6}\right)}}=\frac{2}{\sqrt{3}}$$

$$f'\left(\frac{\pi}{6}\right)=2\,\overset{0}{\cancel{\cos\left(\underset{\frac{\pi}{2}}{3\left(\frac{\pi}{6}\right)}\right)}}\cdot\left(\overset{\downarrow}{-}\overset{1}{\cancel{\sin\left(\underset{\frac{\pi}{2}}{3\left(\frac{\pi}{6}\right)}\right)}}\right)\cdot 3+\overset{\left(\frac{2}{\sqrt{3}}\right)^2}{\cancel{\sec^2\left(\frac{\pi}{6}\right)}}$$

$$=0+\frac{4}{3}=\boxed{\frac{4}{3}}$$ Match!

2(c) $f(x)=\cos(e^{5x}-1)+\sin(\ln(1+6x))$

$$f'(x)=-\sin(e^{5x}-1)\cdot 5e^{5x}+\cos(\ln(1+6x))\cdot\frac{1}{1+6x}\cdot 6$$

$$f'(0)=-\overset{0}{\cancel{\sin(\overset{1}{\cancel{e^{0}}}-1)}}\cdot 5e^{0}+\overset{1}{\cancel{\cos(\overset{0}{\cancel{\ln(1+0)}})}}\cdot\frac{1}{1+0}\cdot 6$$

$$=0+6=\boxed{6}$$ Match!

2(d) $f(x)=e^{\sin(3x)}-\ln(1+\sin(5x))$

$$f'(x)=e^{\sin(3x)}\cdot\cos(3x)\cdot 3-\frac{1}{1+\sin(5x)}\cdot\cos(5x)\cdot 5$$

$$f'(0)=\overset{1}{\cancel{e^{\overset{0}{\cancel{\sin 0}}}}}\cdot\overset{1}{\cancel{\cos 0}}\cdot③-\overset{1}{\cancel{\frac{1}{1+\underset{0}{\cancel{\sin 0}}}}}\cdot\overset{1}{\cancel{\cos 0}}\cdot⑤$$

$$=3-5=\boxed{-2}$$ Match!

3(a) $\int \frac{6-x^7}{x^8}dx = \int \frac{6}{x^8} - \frac{x^7}{x^8}\,dx \overset{\text{prep}}{=} \int 6x^{-8} - \frac{1}{x}\,dx = \frac{6x^{-7}}{-7} - \ln|x| + C = \boxed{\frac{-6}{7x^7} - \ln|x| + C}$

split

Algebra

3(b) $\int \frac{x^7}{6-x^8}\,dx = -\frac{1}{8}\int \frac{1}{u}\,du = -\frac{1}{8}\ln|u| + C = \boxed{-\frac{1}{8}\ln|6-x^8| + C}$

u-sub

$u = 6-x^8$

$du = -8x^7\,dx$

$-\frac{1}{8}du = x^7\,dx$

3(c) $\int \frac{(1+e^{3x})^2}{e^{3x}}\,dx \overset{\text{FOIL}}{=} \int \frac{1+2e^{3x}+e^{6x}}{e^{3x}}\,dx \underset{\text{split}}{=} \int \frac{1}{e^{3x}} + \frac{2\cancel{e^{3x}}}{\cancel{e^{3x}}} + \frac{e^{6x}}{e^{3x}}\,dx$

Algebra

$= \int e^{-3x} + 2 + e^{3x}\,dx = \frac{e^{-3x}}{-3} + 2x + \frac{e^{3x}}{3} + C$

$= \boxed{-\frac{1}{3e^{3x}} + 2x + \frac{e^{3x}}{3} + C}$

☆ K-rule: $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$ $k \neq 0$ constant

3(d) $\int \frac{e^{3x}}{(1+e^{3x})^2}\,dx = \frac{1}{3}\int \overset{u^{-2}}{\frac{1}{u^2}}\,du = \frac{1}{3}\left(\frac{u^{-1}}{-1}\right) + C = \frac{-1}{3u} + C = \boxed{\frac{-1}{3(1+e^{3x})} + C}$

u-sub

$u = 1+e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}du = e^{3x}\,dx$

4(a) $$\int_0^{\ln 3} \frac{1}{e^x(4-e^{-x})}\,dx = \int_3^{11/3} \frac{1}{u}\,du = \ln|u|\Big|_3^{11/3} = \ln|11/3| - \ln|3|$$

$$= \ln\left(\frac{11/3}{\underset{1}{3}}\right) \overset{1/3}{\nearrow}$$

Flip

$$= \boxed{\ln(11/9)}$$

Match!

$u = 4-e^{-x}$

$du = -e^{-x}(-1)\,dx$

$= e^{-x}\,dx$

$du = \frac{1}{e^x}\,dx$

$x=0 \Rightarrow u = 4-\cancelto{1}{e^0} = 3$

$x=\ln 3 \Rightarrow u = 4-e^{-\ln 3}$

$= 4-\cancel{e^{\ln}}(3^{-1})$

$= 4-\frac{1}{3} = 11/3$

4(b) $$\int_e^{e^2} \frac{1}{x(1+\ln x)^2}\,dx = \int_2^3 \frac{1}{u^2}\,\overset{u^{-2}}{du} = \frac{u^{-1}}{-1}\Bigg|_2^3 = -\frac{1}{u}\Bigg|_2^3$$

$$= -\frac{1}{3} \overset{--}{+} \frac{1}{2} = -\frac{2}{6}+\frac{3}{6} = \boxed{\frac{1}{6}}$$

Match!

$u = 1+\ln x$

$du = \frac{1}{x}\,dx$

$x=e \Rightarrow u = 1+\cancelto{1}{\ln e} = 1+1 = 2$

$x=e^2 \Rightarrow u = 1+\cancelto{2}{\ln e^2} = 1+2 = 3$

4(c) $$\int_9^{64} \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx = 2\int_4^9 \frac{1}{\sqrt{u}}\,\overset{u^{-\frac{1}{2}}}{du} = 2\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_4^9 = 4\sqrt{u}\,\Bigg|_4^9$$

$$= 4\left(\cancelto{3}{\sqrt{9}} - \cancelto{2}{\sqrt{4}}\right) = 4(3-2) = \boxed{4}$$

Match!

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$x=9 \Rightarrow u = 1+\cancelto{3}{\sqrt{9}} = 4$

$x=64 \Rightarrow u = 1+\cancelto{8}{\sqrt{64}} = 9$

4(d) $$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^3 x}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^3}\,dx = -\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \overset{u^{-3}}{\frac{1}{u^3}}\,du = \cancel{-}\frac{u^{-2}}{\cancel{-}2}\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} = \frac{1}{2u^2}\Bigg|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$x = \frac{\pi}{6} \Rightarrow u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$x = \frac{\pi}{3} \Rightarrow u = \cos\frac{\pi}{3} = \frac{1}{2}$

$$= \frac{1}{2}\left(\frac{1}{\underbrace{\left(\frac{1}{2}\right)^2}_{\frac{1}{4}}} - \frac{1}{\underbrace{\left(\frac{\sqrt{3}}{2}\right)^2}_{\frac{3}{4}}}\right) = \frac{1}{2}\left(4 - \frac{4}{3}\right)$$

$$= \frac{1}{2}\left(\frac{12}{3} - \frac{4}{3}\right) = \frac{1}{\cancel{2}}\left(\frac{\overset{4}{\cancel{8}}}{3}\right) = \boxed{\frac{4}{3}}$$ Match!

5(a) $$f(x) = 8\ln x - \frac{8}{e^{8x}} + \overset{\text{constant}}{\overset{8}{\cancel{e^{\ln 8}}}} - \frac{8}{x} + \overset{\text{constant}}{\ln 8} + \frac{e^x}{e^{8x}}$$

$$= 8\ln x - 8e^{-8x} + 8 - 8x^{-1} + \ln 8 + e^{-7x}$$

$$f'(x) = 8\left(\frac{1}{x}\right) - 8e^{-8x}(-8) + 0 + 8x^{-2} + 0 + e^{-7x}(-7)$$

$$= \boxed{\frac{8}{x} + \frac{64}{e^{8x}} + \frac{8}{x^2} - \frac{7}{e^{7x}}}$$

5(b) $$g(x) = 8e^x + \frac{1}{e^x} - \overset{x}{\cancel{\ln(e^x)}} - \frac{1}{x^8} - \frac{1}{x} + \left(e^{8x}\cdot e^x\right)$$

$$= 8e^x + e^{-x} - x - x^{-8} - \frac{1}{x} + e^{9x}$$

$$\int g(x)\,dx = 8e^x + \frac{e^{-x}}{-1} - \frac{x^2}{2} \cancel{-} \frac{x^{-7}}{\cancel{-}7} - \ln|x| + \frac{e^{9x}}{9} + C$$

$$= \boxed{8e^x - \frac{1}{e^x} - \frac{x^2}{2} + \frac{1}{7x^7} - \ln|x| + \frac{e^{9x}}{9} + C}$$

6. $\displaystyle\int_{-1}^{2} 2-3x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\cdot\boxed{\left(\frac{3}{n}\right)}$ Always write Formula

$$\left.\begin{array}{l} a=-1 \\ b=2 \end{array}\right\rangle \Delta x=\frac{b-a}{n}=\frac{2-(-1)}{n}=\frac{3}{n}$$

$$x_i = a+i\Delta x = -1+i\left(\frac{3}{n}\right) = -1+\frac{3i}{n}$$

$$= \lim_{n\to\infty} \boxed{\frac{3}{n}} \sum_{i=1}^{n} 2-3\left(-1+\frac{3i}{n}\right)-\left(-1+\frac{3i}{n}\right)^2$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 2+3-\frac{9i}{n}-\left(1-\frac{6i}{n}+\frac{9i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 2+3-\frac{9i}{n}-1+\frac{6i}{n}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty} \boxed{\frac{3}{n}\sum_{i=1}^{n}} 4-\frac{3i}{n}-\frac{9i^2}{n^2}$$ pull all non-i values out of Series Sums

$$= \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} 4 - \frac{3}{n}\sum_{i=1}^{n}\frac{3i}{n} - \frac{3}{n}\sum_{i=1}^{n}\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{12}{n}\overset{n}{\cancel{\sum_{i=1}^{n} 1}} - \frac{9}{n^2}\boxed{\sum_{i=1}^{n} i} - \frac{27}{n^3}\boxed{\sum_{i=1}^{n} i^2}$$

i-Formulas

repartner

$$= \lim_{n\to\infty} \frac{12}{\cancel{n}}\cdot\cancel{n} - \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} 12 - \frac{9}{2}\overset{1}{\cancel{\left(\frac{n}{n}\right)}}\left(\frac{n+1}{n}\right) - \frac{27}{6}\overset{1}{\cancel{\left(\frac{n}{n}\right)}}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty} 12 - \frac{9}{2}(1)\left(\overset{1}{\cancel{\frac{n}{n}}}+\frac{1}{n}\right) - \frac{27}{6}(1)\left(\overset{1}{\cancel{\frac{n}{n}}}+\frac{1}{n}\right)\left(\overset{2}{\cancel{\frac{2n}{n}}}+\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 12 - \frac{9}{2}\left(1+\overset{0}{\cancel{\frac{1}{n}}}\right) - \frac{27}{6}\left(1+\overset{0}{\cancel{\frac{1}{n}}}\right)\left(2+\overset{0}{\cancel{\frac{1}{n}}}\right)$$

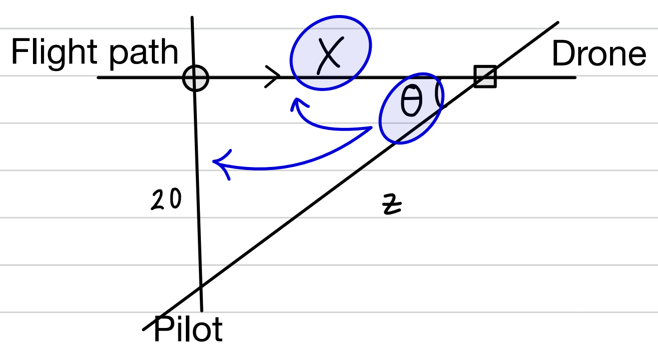
Finally ⟶ Let $n\to\infty$

$$= 12-\frac{9}{2}-\frac{27}{\cancel{6}_3}(1)(\cancel{2}) = 12-\frac{9}{2}-\frac{\cancel{27}^{9}}{\cancel{3}}$$

$$= 12-\frac{9}{2}-9 = 3-\frac{9}{2} = \frac{6}{2}-\frac{9}{2} = \boxed{-\frac{3}{2}}$$ Match!

7. **Diagram**

↳ given



Label All Parts

Variables

Let x = Distance the drone has travelled horizontally

z = Distance between drone and pilot

θ = Angle at the drone corner

Given $\frac{dx}{dt} = 10$ feet/sec

Find $\frac{d\theta}{dt} = ?$ when $z = 40$

Equation

$$\tan\theta = \frac{20}{x}$$

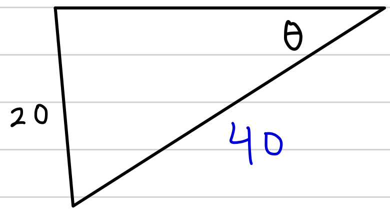
Differentiate

$$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(20x^{-1}\right)$$

$$(\sec\theta)^2 \rightarrow \sec^2\theta\cdot\frac{d\theta}{dt} = -20x^{-2}\cdot\frac{dx}{dt}$$

Extra Solvable Information

$$? \quad \sqrt{(40)^2-(20)^2} = \sqrt{1600-400} = \sqrt{1200} \overset{\text{or}}{=} 20\sqrt{3}$$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{H}{A} = \frac{40}{\sqrt{1200}}$$

Substitute

$$\left(\frac{40}{\sqrt{1200}}\right)^2\cdot\overset{?}{\frac{d\theta}{dt}} = -\frac{20}{\left(\sqrt{1200}\right)^2}\cdot(10)$$

$$\frac{(40)^2}{\left(\sqrt{1200}\right)^2}$$

Solve

$$\frac{d\theta}{dt} = -\frac{20}{\cancel{1200}}(10)\cdot\frac{\cancel{1200}}{1600}$$

$$= \frac{-20\cancel{0}}{160\cancel{0}} = -\frac{1}{8} \text{ Radians/sec.}$$

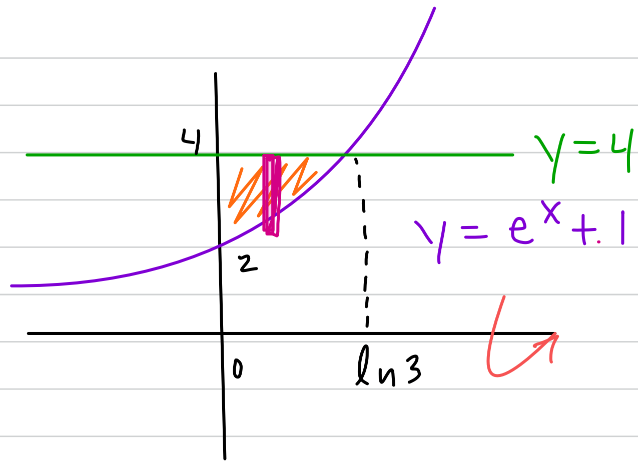
Minus makes sense since Angle shrinking as drone flies

Answer

The Angle at the drone corner is decreasing $\frac{1}{8}$ Radians every second at that Moment

8.

(a)



Intersect?

$e^x + 1 = 4$

$e^x = 3$

$\cancel{\ln}(\cancel{e}^x) = \ln 3$

$\hookrightarrow x = \ln 3$

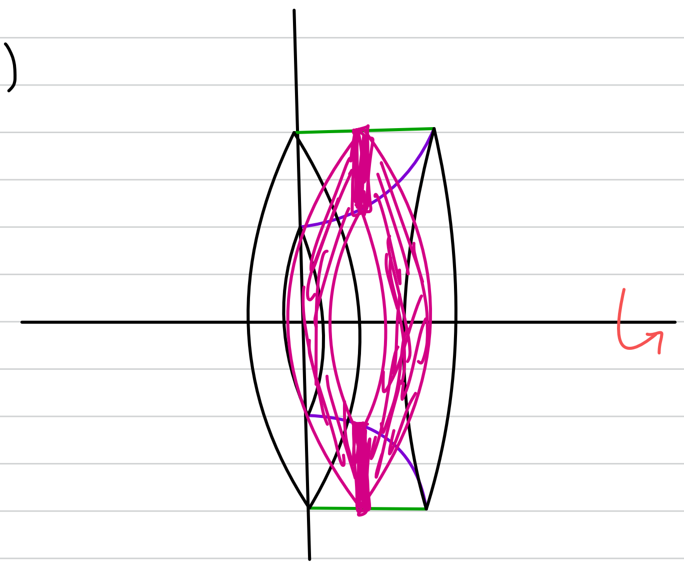
(b) Area = $\int_0^{\ln 3}$ Top − Bottom dx

$= \int_0^{\ln 3} 4 - (e^x + 1)\, dx = \int_0^{\ln 3} 4 - e^x - 1\, dx$

$= \int_0^{\ln 3} 3 - e^x\, dx = 3x - e^x \Big|_0^{\ln 3} = 3\ln 3 - \overset{3}{\cancel{e^{\ln 3}}} - (0 - \overset{1}{\cancel{e^0}})$

$= 3\ln 3 - 3 + 1 = \boxed{3\ln 3 - 2}$

(c)



Washer Method

Volume $= \pi \int_0^{\ln 3}$ (Outer Radius)^2 − (Inner Radius)^2 dx

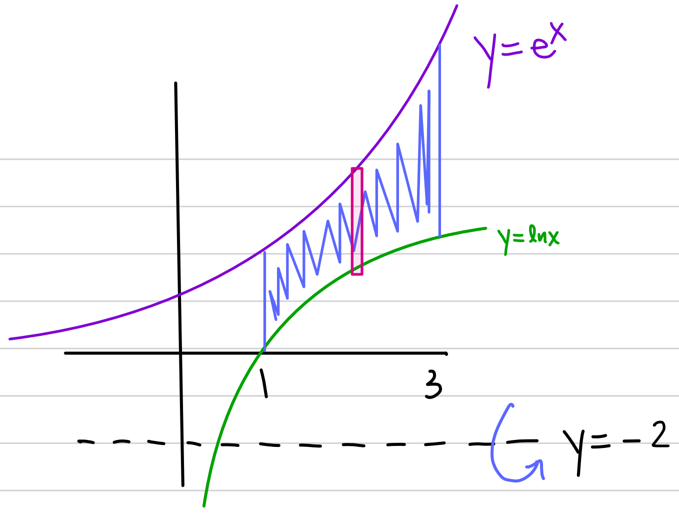
$= \pi \int_0^{\ln 3} 4^2 - (e^x + 1)^2\, dx = \pi \int_0^{\ln 3} 16 - (e^{2x} + 2e^x + 1)\, dx$

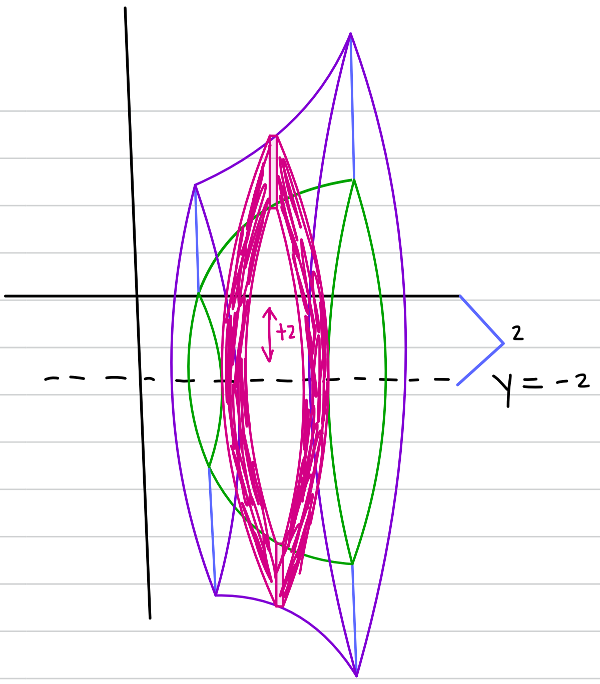
$= \pi \int_0^{\ln 3} 15 - e^{2x} - 2e^x\, dx = \pi\left(15x - \frac{e^{2x}}{2} - 2e^x\right)\Big|_0^{\ln 3}$

$= \pi\left(\left(15\ln 3 - \frac{\overset{3^2}{e^{2\ln 3}}}{2} - 2\cancel{e^{\ln 3}}\right) - \left(0 - \frac{\overset{1}{\cancel{e^0}}}{2} - 2\overset{1}{\cancel{e^0}}\right)\right)$

$= \pi\left(15\ln 3 - \underbrace{\frac{9}{2} - \overbrace{6 + \frac{1}{2}}^{-4} + 2}_{-\frac{8}{2} = -4}\right) = \boxed{\pi\left(15\ln 3 - 8\right)}$

8(d)





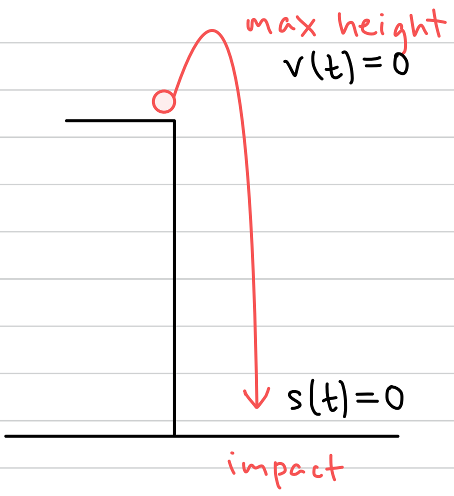
Washer Method

8(e) $\text{Volume} = \pi \int_1^3 (\text{Outer Radius})^2 - (\text{Inner Radius})^2 \, dx$

$$= \pi \int_1^3 (e^x+2)^2 - (\ln x+2)^2 \, dx$$

STOP

9.



$v(0) = +64$ ft/sec

$s(0) = ?$

$v_{impact} = -96$ ft/sec

$a(t) = -32$

$v(t) = -32t + \overset{64}{\cancel{v(0)}} = -32t+64$

$s(t) = -16t^2 + \overset{64}{\cancel{v(0)}}t + \overset{?}{\cancel{s(0)}} = -16t^2+64t+\overset{80}{\cancel{s(0)}}$

Impact: Solve $v(t) = -32t+64 \overset{Set}{=} -96$

$160 = 32t$

↳ $t_{impact} = 5$ seconds

The object hits the ground after 5 seconds.

Plug in $s(5) = -16 \cdot (5)^2 + 64 \cdot (5) + s(0) \overset{Set}{=} 0$

$-16(25) + 320 + s(0) = 0$

Impact ↳ Position = 0

$\underset{-80}{-400 + 320} + s(0) = 0$

↳ $s(0) = 80$

The height of the building is 80 feet

Max Height: Solve $v(t) = -32t+64 \overset{set}{=} 0$

$32t = 64$

↳ $t_{max} = 2$ second

Max Height occurs at time t=2 seconds

$s(t_{max}) = s(2) = -16\underset{4}{(2)^2} + 64(2) + 80$

$= \underset{64}{-64} + 128 + 80 = 144$ feet

Max Height is 144 feet when t= 2 seconds

10. Solution $y(t) = \overset{10}{\cancel{y(0)}}e^{kt} = 10e^{kt}$

Given $y(0) = 10$

$y(3) = 80$

$$\frac{y(3) = 10e^{k\cdot 3}}{10} = \frac{80}{10}$$

$$e^{3k} = 8$$

$$\cancel{\ln(e}^{3k}) = \ln 8$$

$$3k = \ln 8$$

$$k = \frac{\ln 8}{3}$$

OR $k = \frac{1}{3}\ln 8 = \ln(8^{\frac{1}{3}}) = \ln 2$

Plug back in k

$$y(t) = 10e^{\left(\frac{\ln 8}{3}\right)t} = 10e^{\frac{t}{3}\cdot\ln 8} = 10\cancel{e}^{\cancel{\ln}(8^{\frac{t}{3}})} = 10\cdot 8^{\frac{t}{3}}$$

Alternate Formula

OR $y(t) = 10(8^{1/3})^t = 10\cdot 2^t$

$$y(1) = 10\cdot 8^{\frac{1}{3}} = 10\overset{2}{\cancel{\sqrt[3]{8}}} = 20$$

OR $y(1) = 10\cdot 2^1 = 20$

After 1 hour, there were 20 cells.

$$y(12) = 10\cdot 8^{\frac{12}{3}} = 10\cdot 8^4 = 10\cdot(4096) = 40960$$

OR $y(12) = 10\cdot \overset{4096}{2^{12}} = 40{,}960$

After 12 hours, there were 40,960 cells.

$$\frac{y(t) = 10\cdot 8^{\frac{t}{3}}}{10} \overset{\text{set}}{=} \frac{240}{10}$$

$$8^{\frac{t}{3}} = 24$$

$$\ln(8^{\frac{t}{3}}) = \ln(24)$$

$$\frac{t}{3}\ln 8 = \ln(24) \Rightarrow t = \frac{3\ln(24)}{\ln 8}$$

OR $y(t) = 10\cdot 2^t \overset{\text{set}}{=} 240$

$$2^t = 24$$

$$\ln(2^t) = \ln 24$$

$$t\ln 2 = \ln 24$$

$$t = \frac{\ln(24)}{\ln 2}$$

There were 240 cells after $t = \frac{3\ln(24)}{\ln 8}$ hours

Note $\frac{3\ln(24)}{\ln(2^3)} = \frac{\cancel{3}\ln(24)}{\cancel{3}\ln 2}$ SAME