Math 106 Final May 9, 2025

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• Numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [24 Points] Compute each of the following Derivatives.

(a) $\frac{d}{dx}\ln\left(\frac{\sqrt{8-x^5}\cdot\tan(e^x)}{(1-e^x)^7\cdot e^{-4x}}\right)$ Do not simplify answer.

(b) Let $y=5^x$. Show that $\frac{dy}{dx}=5^x(\ln 5)$ using Logarithmic Differentiation.

(c) Find the Local Maximum and/or Local Minimum **Values** for $f(x)=x^2e^x$.

(d) At what **Point** (x,y) on the curve $y=[\ln(x+4)]^2$ is the Tangent Line **Horizontal**?

2. [20 Points] Compute the following Derivatives. Simplify.

(a) Consider $f(x)=2\sin(5x)+\sin(6x)-5\cos(2x)-\sin(3x)$ Show that $f'\left(\frac{\pi}{6}\right)=\boxed{-6}$

(b) Consider $f(x)=\cos^2(3x)+\tan x \overset{\text{prep}}{=} (\cos(3x))^2+\tan x$. Show that $f'\left(\frac{\pi}{6}\right)=\boxed{\frac{4}{3}}$

(c) Consider $f(x)=\cos\left(e^{5x}-1\right)+\sin\left(\ln(1+6x)\right)$ Show that $f'(0)=\boxed{6}$

(d) Consider $f(x)=e^{\sin(3x)}-\ln(1+\sin(5x))$ Show that $f'(0)=\boxed{-2}$

3. [28 Points] Compute each of the following Integrals.

(a) $\int\frac{6-x^7}{x^8}\,dx$ (b) $\int\frac{x^7}{6-x^8}\,dx$ (c) $\int\frac{(1+e^{3x})^2}{e^{3x}}\,dx$ (d) $\int\frac{e^{3x}}{(1+e^{3x})^2}\,dx$

4. [40 Points] Compute the following Definite Integrals. Match the given answer.

(a) $\int_0^{\ln 3}\frac{1}{e^x(4-e^{-x})}\,dx=\boxed{\ln\left(\frac{11}{9}\right)}$ (b) $\int_e^{e^2}\frac{1}{x(1+\ln x)^2}\,dx=\boxed{\frac{1}{6}}$

(c) $\int_9^{64}\frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx=\boxed{4}$ (d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x}{\cos^3 x}\,dx \overset{\text{hint}}{=}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x}{(\cos x)^3}\,dx=\boxed{\frac{4}{3}}$

5. [14 Points] (a) Consider $f(x)=8\ln x-\frac{8}{e^{8x}}+e^{\ln 8}-\frac{8}{x}+\ln 8+\frac{e^x}{e^{8x}}$ Compute $f'(x)$.

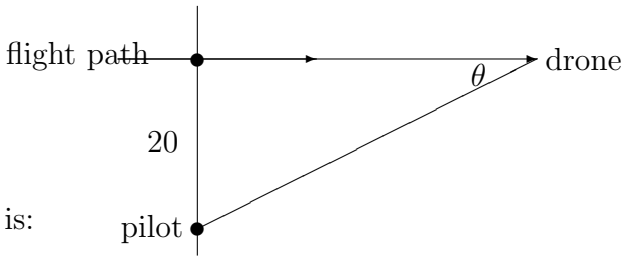
(b) Consider $g(x)=8e^x+\frac{1}{e^x}-\ln(e^x)-\frac{1}{x^8}-\frac{1}{x}+\left(e^{8x}\cdot e^x\right)$ Compute $\int g(x)\,dx$.

6. [16 Points] Show that $\int_{-1}^{2} 2-3x-x^2\,dx = \boxed{-\frac{3}{2}}$ using **ONLY** the **Limit Definition of the Definite Integral**

7. [12 Points] A drone is flying exactly 20 feet above its pilot on the ground. Suppose that the drone is flying horizontally at 10 feet per second. Consider the Angle θ as shown in the diagram. How fast is this Angle θ changing when the (diagonal) distance between the pilot and the drone is 40 feet?

• Diagram

The picture at arbitrary time t is:



8. [24 Points] (a) Consider the region bounded by $y = e^x + 1$, $y = 4$, and $x = 0$. Sketch and shade the bounded region.

(b) **COMPUTE** the **Area** bounded in part (a).

(c) **COMPUTE** the **Volume** of the three-dimensional solid obtained by rotating the region in (a) about the ***x*-axis**. Sketch the solid, along with one of the approximating *Washers*.

(d) Consider a **different** region bounded by $y = e^x$, $y = \ln x$, $x = 1$ and $x = 3$. Sketch and shade the bounded region.

(e) **Set-Up** but **DO NOT EVALUATE** the integral to compute the **Volume** of the three-dimensional solid obtained by rotating the region in (d) about the line $\boldsymbol{y = -2}$. Sketch the solid, along with one of the approximating *Washers*.

9. [10 Points] Use acceleration $a(t) = -32$ feet per second squared.

Mark throws a rock straight upwards from the top of a building with Initial Velocity of 64 feet per second. The ball hits the ground with an Impact Velocity of -96 feet per second. Draw a sketch.

• How tall is the building? Hint: First find the time at impact $t_{impact}\ldots$

• After how many seconds is the Maximum Height reached?

• What is the Maximum Height reached?

Hints: $(16)(25) = 400$ and $(64)(5) = 320$ and $(32)(5) = 160$

10. [12 Points] A population of bacteria was growing exponentially. Initially there were 10 cells. After 3 hours there were 80 cells.

• How many cells were there after 1 hour? Simplify. Hint: $8^{\frac{1}{3}} = \sqrt[3]{8}$

• How many cells were there after 12 hours? Hint: $8^4 = 4096$.

• When were there 240 cells? (You can leave this *time t* answer in terms of logs)