Math 106 Final Exam Spring 2023

1(a) $y = (\ln x)^{\ln x}$

$\ln y = \ln\left((\ln x)^{\ln x}\right)$

$\ln y = \ln x \cdot \ln(\ln x)$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\ln x \cdot \ln(\ln x)\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cancel{\ln x} \cdot \frac{1}{\cancel{\ln x}} \cdot \frac{1}{x} + \ln(\ln x) \cdot \frac{1}{x}$

$\frac{dy}{dx} = \cancel{y}\ (\ln x)^{\ln x} \left(\frac{1}{x} + \frac{\ln(\ln x)}{x} \right)$

$= \boxed{(\ln x)^{\ln x} \left(\frac{1}{x} + \frac{\ln(\ln x)}{x} \right)}$

1(b) $\frac{d}{dx} \ln\left(\frac{(6-x^4)^8 \cdot e^{\tan x}}{\sqrt{9-x^2} \cdot e^{-6x}} \right)$ First use Log Algebra

$= \frac{d}{dx} \ln\left((6-x^4)^8 \cdot e^{\tan x}\right) - \ln\left(\sqrt{9-x^2} \cdot e^{-6x}\right)$

$= \frac{d}{dx} \ln\left((6-x^4)^8\right) + \cancel{\ln} e^{\tan x} - \left(\ln\left((9-x^2)^{1/2}\right) + \cancel{\ln(e}^{-6x})\right)$ $-6x$

$= \frac{d}{dx}\ 8 \ln(6-x^4) + \tan x - \frac{1}{2}\ln(9-x^2) + 6x$

$= \boxed{8 \cdot \frac{1}{6-x^4} \cdot (-4x^3) + \sec^2 x \cancel{-} \frac{1}{\cancel{2}} \cdot \frac{1}{9-x^2} \cdot (\cancel{-2}x) + 6}$

OR $= -\frac{32x^3}{6-x^4} + \sec^2 x + \frac{x}{9-x^2} + 6$

1(c) $f(x) = \frac{x^4}{e^x}$

$$f'(x) = \frac{e^x(4x^3) - x^4 \cdot e^x}{(e^x)^2} = \frac{\cancel{e^x}\, x^3(4-x)}{e^{\cancel{2x}}} = \frac{x^3(4-x)}{\underset{+}{e^x}} \overset{\text{set}}{=} 0$$

$\Rightarrow x^3 = 0$ or $4 - x = 0$

$\hookrightarrow x = 0$ $\quad\hookrightarrow x = 4$ Critical Numbers

Sign Testing into First Derivative

	-1	0	1	4	10
f'	$-$		$+$		$-$
f	$\searrow$	Min	$\nearrow$	Max	$\searrow$

$f(0) = \frac{0}{e^0} = \frac{0}{1} = 0$ Local Minimum Value

$f(4) = \frac{4^4}{e^4} = \frac{256}{e^4}$ Local Maximum Value

1(d) Let $y = \ln x$

Invert $e^y = \cancel{e}^{\ln x}$

$e^y = x$

Differentiate

$$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$$

$$e^y \cdot \frac{dy}{dx} = 1$$

Solve

$$\frac{dy}{dx} = \frac{1}{\cancel{e^y} \to x} = \frac{1}{x} \checkmark$$

2(a) $f(x) = \cos(7x) + \cos(6x) + \underbrace{\cancel{\sin\left(\frac{\pi}{3}\right)}}_{\text{constant}}^{\frac{\sqrt{3}}{2}} + \sin(3x) + \sin(4x)$

$$f'(x) = -7\sin(7x) - 6\sin(6x) + 0 + 3\cos(3x) + 4\cos(4x)$$

$$f'\left(\frac{\pi}{6}\right) = -7\overbrace{\cancel{\sin\left(\frac{7\pi}{6}\right)}}^{-\frac{1}{2}} - 6\overbrace{\cancel{\sin\left(\cancel{6}\cdot\frac{\pi}{\cancel{6}}\right)}}^{0} + 3\overbrace{\cancel{\cos\left(\cancel{3}\cdot\frac{\pi}{\cancel{6}_2}\right)}}^{0} + 4\overbrace{\cancel{\cos\left(\frac{\cancel{4}^2\pi}{\cancel{6}_3}\right)}}^{-\frac{1}{2}}$$

$$= \frac{7}{2} - 0 + 0 - \frac{4}{2} = \frac{3}{2}$$

Match!

2(b) $f(x) = \cos(\ln(1+4x)) - \cancelto{0}{\ln 1} - \ln(1+\cos(5x)) - e^{\tan(6x)} - \sin(e^{7x}-1)$

$$f'(x) = -\sin(\ln(1+4x)) \cdot \frac{1}{1+4x} \cdot 4 - \frac{1}{1+\cos(5x)} \cdot (-\sin(5x)) \cdot 5 - e^{\tan(6x)} \cdot \sec^2(6x) \cdot 6 - \cos(e^{7x}-1) e^{7x} \cdot 7$$

$$f'(0) = -\cancelto{0}{\sin(\cancelto{0}{\ln 1})} \cdot \frac{1}{1} \cdot 4 - \cancelto{0}{\cancelto{-\frac{1}{2}}{\frac{1}{1+\cos 0}} \cdot (-\cancelto{0}{\sin 0}) \cdot 5} - \cancelto{6}{\cancelto{1}{e^{\tan 0}} \cdot \cancelto{1}{\sec^2 0} \cdot 6} - \cancelto{7}{\cancelto{1}{\cos(\cancelto{1}{e^0}-1)}\, \cancelto{1}{e^0} \cdot 7}$$

$$= 0 - 0 - 6 - 7 = -13$$

3(a) $$\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sec^2 u\,du = 2\tan u + C = 2\tan\sqrt{x} + C$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

3(b) $$\int \frac{x^3}{(9-x^2)^2}\,dx = \int \frac{x^2\,x}{(9-x^2)^2}\,dx = -\frac{1}{2}\int \frac{9-u}{u^2}\,du = -\frac{1}{2}\int \frac{9}{u^2} - \frac{u}{u^2}\,du$$

invert

$u = 9 - x^2 \Rightarrow x^2 = 9 - u$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$$= -\frac{1}{2}\int 9u^{-2} - \frac{1}{u}\,du = -\frac{1}{2}\left(\frac{9u^{-1}}{-1} - \ln|u|\right) + C$$

$$= -\frac{1}{2}\left(-\frac{9}{u} - \ln|u|\right) + C$$

$$= \boxed{-\frac{1}{2}\left(-\frac{9}{9-x^2} - \ln|9-x^2|\right) + C}$$

FOIL

3(c) $$\int \left(e^{3x} + \frac{1}{e^{2x}}\right)\left(e^x + \frac{1}{e^{5x}}\right) dx = \int e^{4x} + \frac{e^{3x}}{e^{5x}} + \frac{e^x}{e^{2x}} + \frac{1}{e^{7x}}\,dx$$

$$= \int e^{4x} + e^{-2x} + e^{-x} + e^{-7x}\,dx$$

$$= \frac{e^{4x}}{4} + \frac{e^{-2x}}{-2} + \frac{e^{-x}}{-1} + \frac{e^{-7x}}{-7} + C$$

$$= \boxed{\frac{e^{4x}}{4} - \frac{1}{2e^{2x}} - \frac{1}{e^x} - \frac{1}{7e^{7x}} + C}$$

3(d) $\displaystyle\int \frac{\sin x}{e^{\cos x}}\,dx = -\int \frac{1}{e^u}\,du = -\int e^{-u}\,du = \cancel{-}\frac{e^{-u}}{\cancel{-}1} + c = \frac{1}{e^u} + c = \boxed{\frac{1}{e^{\cos x}} + c}$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \\ -du &= \sin x\,dx \end{aligned}}$$

4(a) $\displaystyle\int_{\ln 2}^{\ln 4} \frac{1}{e^{2x}(1-e^{-2x})}\,dx = \frac{1}{2}\int_{3/4}^{15/16} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_{3/4}^{15/16} = \frac{1}{2}\left(\ln\left(\frac{15}{16}\right) - \ln\left(\frac{3}{4}\right)\right)$

$$= \frac{1}{2}\ln\left(\frac{\frac{15}{16}\,\frac{4}{3}}{\frac{3}{4}}\right) = \frac{1}{2}\ln\left(\frac{5}{4}\right)$$

$$= \ln\left(\left(\frac{5}{4}\right)^{1/2}\right) = \ln\left(\frac{\sqrt{5}}{\sqrt{4}}\right) = \boxed{\ln\left(\frac{\sqrt{5}}{2}\right)}$$ Match

$$\boxed{\begin{aligned} u &= 1-e^{-2x} \\ du &= 2e^{-2x}dx \\ \frac{1}{2}du &= \frac{1}{e^{2x}}dx \end{aligned}}$$

$$\boxed{\begin{aligned} &x=\ln 2 \Rightarrow u = 1-e^{-2\ln 2} = 1-\cancel{e}^{\ln(2^{-2})} = 1-\frac{1}{4} = \frac{3}{4} \\ &x=\ln 4 \Rightarrow u = 1-e^{-2\ln 4} = 1-\cancel{e}^{\ln(4^{-2})} = 1-\frac{1}{16} = \frac{15}{16} \end{aligned}}$$

4(b) $\displaystyle\int_{e^2}^{e^7} \frac{8}{x\sqrt{2+\ln x}}\,dx = 8\int_4^9 \frac{1}{\sqrt{u}}\,du = 8\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Big|_4^9 = 16\sqrt{u}\Big|_4^9$

($u^{-\frac{1}{2}}$)

$$= 16\left(\cancel{\sqrt{9}}^{\,3} - \cancel{\sqrt{4}}^{\,2}\right) = 16$$ Match

$$\boxed{\begin{aligned} u &= 2+\ln x \\ du &= \frac{1}{x}dx \end{aligned}}$$

$$\boxed{\begin{aligned} &x=e^2 \Rightarrow u = 2+\cancel{\ln e}^2 = 4 \\ &x=e^7 \Rightarrow u = 2+\cancel{\ln e}^7 = 9 \end{aligned}}$$

4(c) $\displaystyle\int_1^3 \frac{1}{x^2}\sin\left(\frac{\pi}{x}\right)dx = -\frac{1}{\pi}\int_{\pi}^{\frac{\pi}{3}} \sin u\,du = \cancel{-}\frac{1}{\pi}(\cancel{-}\cos u)\Big|_{\pi}^{\frac{\pi}{3}}$

$$= \frac{1}{\pi}\left(\cancel{\cos\frac{\pi}{3}}^{\,\frac{1}{2}} - \cancel{\cos\pi}^{\,-1}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2}+1\right) = \frac{3}{2\pi}$$

$$\boxed{\begin{aligned} u &= \frac{\pi}{x} \\ du &= -\frac{\pi}{x^2}dx \\ -\frac{1}{\pi}du &= \frac{1}{x^2}dx \end{aligned}}$$

$$\boxed{\begin{aligned} &x=1 \Rightarrow u=\pi \\ &x=3 \Rightarrow u=\frac{\pi}{3} \end{aligned}}$$

4(d) $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan^3 x}\,dx = \int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{(\tan x)^3}\,dx = \int_1^{\sqrt{3}} \frac{1}{u^3}\,du = \int_1^{\sqrt{3}} u^{-3}\,du = \left.\frac{u^{-2}}{-2}\right|_1^{\sqrt{3}} = \left.-\frac{1}{2u^2}\right|_1^{\sqrt{3}}$

$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x\,dx \end{aligned}}$ $\boxed{\begin{aligned} x = \tfrac{\pi}{4} &\Rightarrow u = \tan\tfrac{\pi}{4} = 1 \\ x = \tfrac{\pi}{3} &\Rightarrow u = \tan\tfrac{\pi}{3} = \sqrt{3} \end{aligned}}$

$= -\frac{1}{2}\left(\frac{1}{3} - 1\right) = -\frac{1}{\cancel{2}}\left(\frac{-2}{3}\right) = \frac{1}{3}$

Match!

4(e) $\displaystyle\int_{\pi/6}^{\pi/3} \tan x\,dx = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\cos x}\,dx = -\int_{\sqrt{3}/2}^{1/2} \frac{1}{u}\,du = -\ln|u|\Big|_{\sqrt{3}/2}^{1/2}$

$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \\ -du &= \sin x\,dx \end{aligned}}$ $\boxed{\begin{aligned} x = \tfrac{\pi}{6} &\rightarrow u = \cos\tfrac{\pi}{6} = \tfrac{\sqrt{3}}{2} \\ x = \tfrac{\pi}{3} &\rightarrow u = \cos\tfrac{\pi}{3} = \tfrac{1}{2} \end{aligned}}$

$= -\left(\ln\left|\frac{1}{2}\right| - \ln\left|\frac{\sqrt{3}}{2}\right|\right) = \ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right)$

$= \ln\left(\frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}}\right) = \ln\sqrt{3} = \ln\left(3^{1/2}\right) = \frac{1}{2}\ln 3$

Match!

5. $f(x) = e^{7x} - \frac{1}{e^{7x}} + \underbrace{e^{e^{e^7}}}_{\text{constant}} + x^e + \underbrace{\ln(\cancel{\ln}\,\cancel{e}^{7})}_{\text{constant}} + \frac{7}{x} - \frac{1}{e^x} + 7e^x + \frac{e^x}{e^{7x}}$

prep

$= e^{7x} - e^{-7x} + \underbrace{e^{e^{e^7}}}_{\text{constant}} + x^e + \underbrace{\ln 7}_{\text{constant}} + 7x^{-1} - e^{-x} + 7e^x + e^{-6x}$

(a) $f'(x) = \boxed{7e^{7x} + 7e^{-7x} + 0 + e\,x^{e-1} + 0 - 7x^{-2} + e^{-x} + 7e^x - 6e^{-6x}}$

(b) $\int f(x)\,dx = \boxed{\frac{e^{7x}}{7} - \frac{e^{-7x}}{-7} + e^{e^{e^7}}\cdot x + \frac{x^{e+1}}{e+1} + (\ln 7)x + 7\ln|x| - \frac{e^{-x}}{-1} + 7e^x + \frac{e^{-6x}}{-6} + C}$

OR $\ln(\ln(e^7))\cdot x$

6 (a) $\displaystyle\int_{-1}^{2} 2 - 2x - x^2\,dx = 2x - x^2 - \frac{x^3}{3}\Big|_{-1}^{2} = \cancel{4} - \cancel{4} - \frac{8}{3} - \left(\underbrace{-2 - 1}_{-3} + \frac{1}{3}\right)$

Definite Integral

$= -\frac{8}{3} + 3 - \frac{1}{3} = 3 - \frac{\cancel{9}^{\,3}}{\cancel{3}} = 0$ Match!

$$6\,(b)\quad \int_{-1}^{2} 2-2x-x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Limit Definition

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\cdot\frac{3}{n}$$

$a=-1 \quad b=2$

$$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$-\left(1-\frac{6i}{n}+\frac{9i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 2-2\left(-1+\frac{3i}{n}\right)-\left(-1+\frac{3i}{n}\right)^2$$

$x_i = a + i\Delta x$

$= -1 + \frac{3i}{n}$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 2+2-\cancel{\frac{6i}{n}}-1+\cancel{\frac{6i}{n}}-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 3-\frac{9i^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} 3 - \frac{3}{n} \sum_{i=1}^{n} \frac{9}{n^2} i^2$$

$$= \lim_{n\to\infty} \frac{9}{n} \cancel{\sum_{i=1}^{n} 1}^{\,n} - \frac{27}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{9}{\cancel{n}}\cdot\cancel{n} - \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$n\cdot n\cdot n$

$$= \lim_{n\to\infty} 9 - \frac{27}{6}\cdot\left(\cancel{\frac{n}{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$ split-split

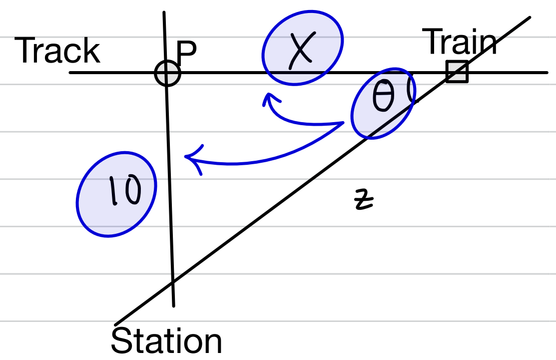
$$= \lim_{n\to\infty} 9 - \frac{27}{6}\cdot 1\cdot \cancel{\left(1+\cancel{\frac{1}{n}}^{\,0}\right)}^{\,1}\cancel{\left(2+\cancel{\frac{1}{n}}^{\,0}\right)}^{\,2}$$

$\infty \quad \infty$

$$= \lim_{n\to\infty} 9 - \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,\cancel{3}}}\cdot 1\cdot 1\cdot \cancel{2} = 9-9 = 0$$ Match

7. Diagram

given



Label All Parts

Variables

Let x = Distance between train and Point P

z = Distance between train and station

θ = Angle between track and line connecting the Train the Station

Given $\frac{dx}{dt} = 6$ ft/sec

Find $\frac{d\theta}{dt} = ?$ when $z = 20$

Equation

$$\tan\theta = \frac{10}{x}$$

Differentiate

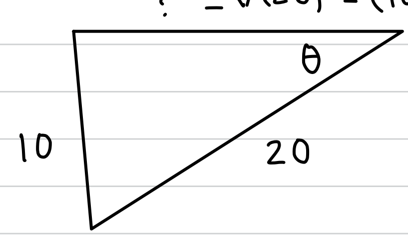
$$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(10x^{-1}\right)$$

$(\sec\theta)^2$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -10x^{-2} \cdot \frac{dx}{dt}$$

Extra Solvable Information

$$? = \sqrt{(20)^2 - (10)^2} = \sqrt{400 - 100} = \sqrt{300} \overset{\text{OR}}{=} \sqrt{100}\sqrt{3} = 10\sqrt{3}$$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{H}{A} = \frac{20}{\sqrt{300}}$$

Substitute

?

$$\left(\frac{20}{\sqrt{300}}\right)^2 \cdot \frac{d\theta}{dt} = -\frac{10}{(\sqrt{300})^2} \cdot 6$$

Solve

$$\frac{d\theta}{dt} = -\frac{60}{3\cancel{00}} \cdot \frac{(\sqrt{\cancel{300}})^2}{(20)^2} \quad \frac{\cancel{300}}{400}$$

$$= -\frac{60}{400} = -\frac{6}{40} = -\frac{3}{20}$$

Radians per Second

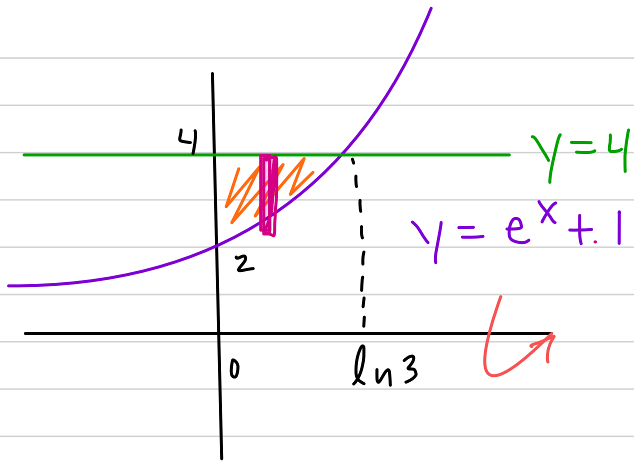
negative, decreasing makes sense

Answer

This angle is <u>decreasing</u> at a rate of $\frac{3}{20}$ Radians every second at this Moment.

8.

(a)



Intersect?

$e^x+1=4$

$e^x=3$

$\cancel{\ln}(\cancel{e}^x)=\ln 3$

$\hookrightarrow x=\ln 3$

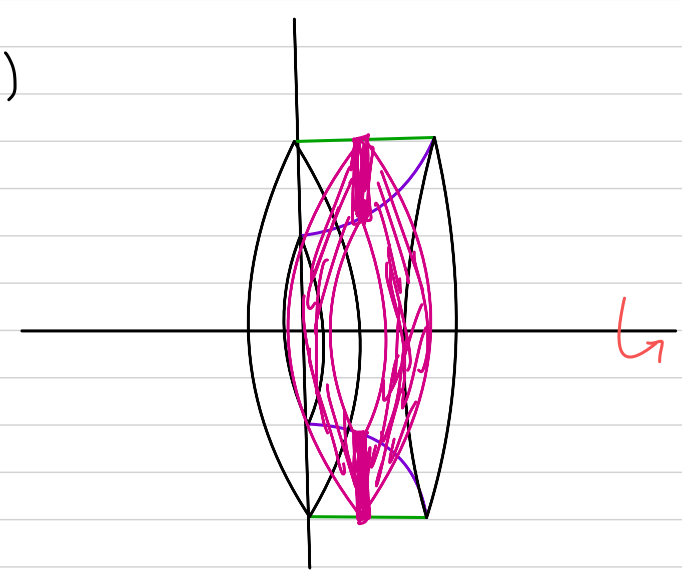
(b) Area $= \int_0^{\ln 3} \text{Top} - \text{Bottom}\, dx$

$= \int_0^{\ln 3} 4-(e^x+1)\,dx = \int_0^{\ln 3} 4-e^x-1\,dx$

$= \int_0^{\ln 3} 3-e^x\,dx = 3x-e^x \Big|_0^{\ln 3} = 3\ln 3 - \cancel{e^{\ln 3}}^{\,3} - (0-\cancel{e^0}^{\,1})$

$= 3\ln 3 - 3 + 1 = \boxed{3\ln 3 - 2}$

(c)



Volume $= \pi \int_0^{\ln 3} (\text{Outer Radius})^2 - (\text{Inner Radius})^2\, dx$

$= \pi \int_0^{\ln 3} 4^2-(e^x+1)^2\,dx = \pi \int_0^{\ln 3} 16-(e^{2x}+2e^x+1)\,dx$

$= \pi \int_0^{\ln 3} 15-e^{2x}-2e^x\,dx = \pi\left(15x-\frac{e^{2x}}{2}-2e^x\right)\Big|_0^{\ln 3}$

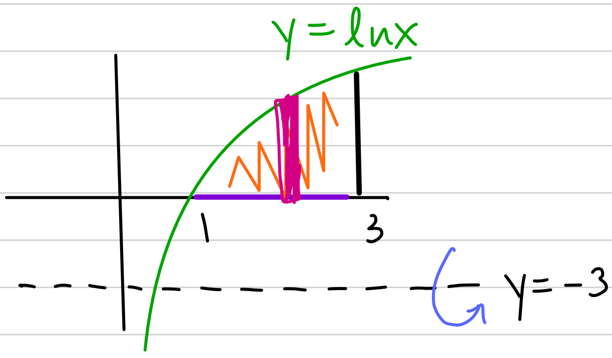
$= \pi\left(\left(15\ln 3 - \frac{e^{2\ln 3}}{2} - 2\cancel{e^{\ln 3}}\right) - \left(0-\frac{\cancel{e^0}^{\,1}}{2}-2\cancel{e^0}^{\,1}\right)\right)$, $\quad 3^2$

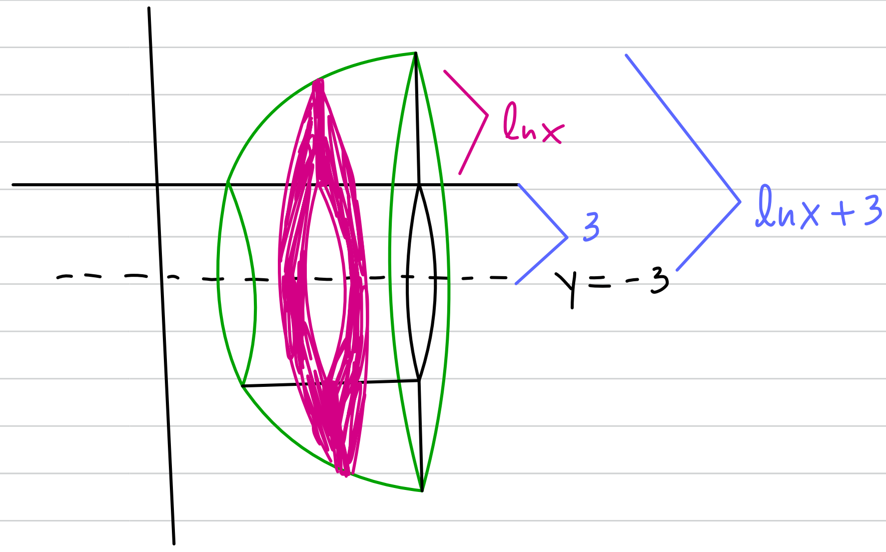
$= \pi\left(15\ln 3 - \frac{9}{2} - 6 + \frac{1}{2} + 2\right) = \boxed{\pi(15\ln 3 - 8)}$

-4

$-\frac{8}{2}=-4$

8(d)



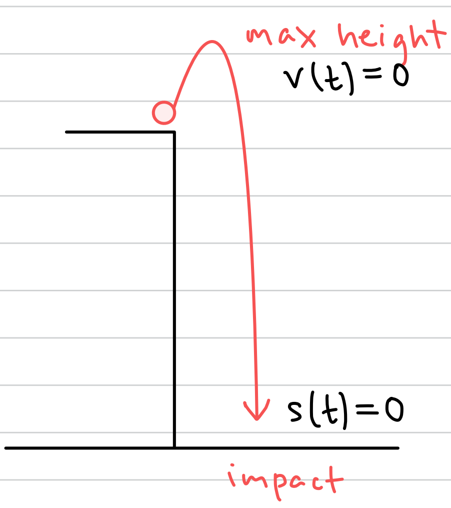


8(e) $\text{Volume} = \pi \int_1^3 (\text{Outer Radius})^2 - (\text{Inner Radius})^2 \, dx$

$$= \boxed{\pi \int_1^3 \left((\ln x)+3\right)^2 - (3)^2 \, dx}$$

STOP

9.



$v(0) = +32$ ft/sec

$s(0) = ?$

$v_{impact} = -64$ ft/sec

$a(t) = -32$

$v(t) = -32t + \cancelto{32}{v(0)} = -32t + 32$

$s(t) = -16t^2 + \cancelto{32}{v(0)}t + \cancelto{?}{s(0)} = -16t^2 + 32t + \cancelto{48}{s(0)}$

Impact: Solve $v(t) = -32t + 32 \overset{\text{set}}{=} -64$

$96 = 32t$

$t_{impact} = 3$ seconds

The ball hits the ground at $t = 3$ seconds.

Plug in $s(3) = -16 \cdot (3)^2 + 32 \cdot (3) + s(0) \overset{\text{set}}{=} 0$ Impact ↳ Position = 0

$-16 \cdot 9 + 96 + s(0) = 0$

$-144 + 96 + s(0) = 0$

$s(0) = 48$ feet

The Height of Building is 48 feet.

Max Height: Solve $v(t) = -32t + 32 \overset{\text{set}}{=} 0$

$32t = 32$

$t_{max} = 1$ second

$s(t_{max}) = s(1) = -16 + 32 + 48 = 64$ feet

The Maximum Height is 64 feet when $t = 1$ second.

10. Solution $y(t) = \cancel{y(0)}^{\,3} e^{kt} = 3e^{kt}$

Given $y(0) = 3$

$y(2) = 12 \hookrightarrow y(2) = \frac{3e^{k \cdot 2}}{3} \overset{set}{=} \frac{12}{3}$

$e^{2k} = 4$

$\cancel{\ln}(\cancel{e}^{2k}) = \ln 4$

$2k = \ln 4$

$k = \frac{\ln 4}{2} = \frac{1}{2} \ln 4 = \ln(4^{1/2}) \overset{OR}{=} \ln 2$

Plug back in k

$y(t) = 3e^{(\ln 2) \cdot t} = 3e^{t \cdot \ln 2} = 3\cancel{e}^{\ln(2^t)} = 3 \cdot 2^t \quad \underline{\underline{OR}} \quad 3 \cdot 4^{t/2}$

After 6 hours there are 192 cells.

$y(6) = 3 \cdot 2^6 = 192$ cells

$\underline{\underline{OR}} = 3 \cdot 4^{6/2} = 3 \cdot 4^3 = 3 \cdot 64 = 192$

$2^4 = 16$
$2^5 = 32$
$2^6 = 64$

$64 \times 3 = 192$

$\boxed{\text{There are 999 cells after } t = \frac{\ln(333)}{\ln 2} \text{ hours}}$

$y(t) = 3 \cdot 2^t \overset{set}{=} 999$

$\Rightarrow 2^t = 333 \Rightarrow \ln(2^t) = \ln(333)$

$t \cdot \ln 2 = \ln(333) \qquad t = \frac{\ln(333)}{\ln 2}$ hours

$\underline{\underline{OR}}$

$3 \cdot 4^{t/2} \overset{set}{=} 999$

$4^{t/2} = 333$

$\ln(4^{t/2}) = \ln(333)$

$\frac{t}{2} \ln 4 = \ln(333)$

$t = \frac{2\ln(333)}{\ln 4}$ Alternate Answer

10(b) Solution $y(t) = \cancel{y(0)}^{\,10{,}000} e^{kt} = 10{,}000\, e^{kt}$

Given $y(0) = 10{,}000$

$y(3) = 8{,}000 \hookrightarrow y(3) = 10{,}000\, e^{3k} \overset{set}{=} 8{,}000$

$e^{3k} = \frac{\cancel{8000}}{\cancel{10\,000}} = \frac{4}{5}$

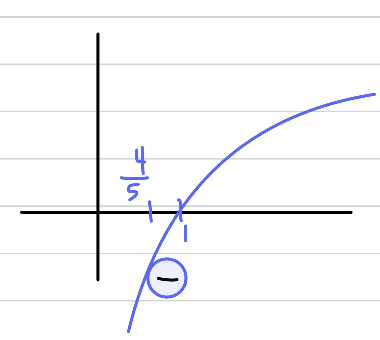
$\cancel{\ln}(\cancel{e}^{3k}) = \ln\left(\frac{4}{5}\right)$

$3k = \ln\left(\frac{4}{5}\right)$

$k = \frac{\ln\left(\frac{4}{5}\right)}{3}$

$y(t) = 10{,}000\, e^{\frac{\ln\left(\frac{4}{5}\right)}{3} \cdot t}$

$= 10{,}000\, \cancel{e}^{\ln\left(\left(\frac{4}{5}\right)^{t/3}\right)} = 10{,}000 \cdot \left(\frac{4}{5}\right)^{\frac{t}{3}}$



Note: $k < 0$ since

$\ln\left(\frac{4}{5}\right) < 0$ b/c

$\frac{4}{5} < 1$ and $\ln 1 = 0$

and Log increasing

↳ Decay

$y(9) = 10{,}000 \cdot \left(\frac{4}{5}\right)^{\frac{9}{3}} = 10{,}000 \left(\frac{4}{5}\right)^3 \overset{given}{=} 5{,}120$

$\boxed{\text{After 9 years the car will be valued at \$5,120.}}$