Extra Riemann Sum, Definite Integral Example

Evaluate $\int_{-2}^{3} x^2-4x+3\,dx$ using Riemann Sums and the limit definition of the definite integral.

Here $f(x)=x^2-4x+3,\quad a=-2,\quad b=3,\quad \Delta x=\dfrac{b-a}{n}=\dfrac{3-(-2)}{n}=\dfrac{5}{n}$
and $x_i=a+i\Delta x=-2+i\left(\dfrac{5}{n}\right)=-2+\dfrac{5i}{n}$.

$$\begin{aligned}
\int_{-2}^{3} x^2-4x+3\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+\frac{5i}{n}\right)\left(\frac{5}{n}\right)\\
&= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^{n}\left(-2+\frac{5i}{n}\right)^2-4\left(-2+\frac{5i}{n}\right)+3\\
&= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^{n}\left(4-\frac{20i}{n}+\frac{25i^2}{n^2}+8-\frac{20i}{n}+3\right)\\
&= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^{n}\left(\frac{25i^2}{n^2}-\frac{40i}{n}+15\right)\\
&= \lim_{n\to\infty}\frac{5}{n}\sum_{i=1}^{n}\frac{25i^2}{n^2}-\frac{5}{n}\sum_{i=1}^{n}\frac{40i}{n}+\frac{5}{n}\sum_{i=1}^{n}15\\
&= \lim_{n\to\infty}\frac{125}{n^3}\sum_{i=1}^{n}i^2-\frac{200}{n^2}\sum_{i=1}^{n}i+\frac{75}{n}\sum_{i=1}^{n}1\\
&= \lim_{n\to\infty}\frac{125}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)-\frac{200}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{75}{\cancel{n}}(\cancel{n})\\
&= \lim_{n\to\infty}\frac{125}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)-\frac{200}{2}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)+75\\
&= \lim_{n\to\infty}\frac{125}{6}(1)\left(1+\cancelto{0}{\frac{1}{n}}\right)\left(2+\cancelto{0}{\frac{1}{n}}\right)-(100)(1)\left(1+\cancelto{0}{\frac{1}{n}}\right)+75\\
&= \frac{125}{6}(1)(1)(2)-(100)(1)(1)+75=\frac{125}{3}-100+75\\
&= \frac{125}{3}-100+75=\frac{125}{3}-25=\frac{125}{3}-\frac{75}{3}=\boxed{\frac{50}{3}}
\end{aligned}$$