Math 105, Fall 2013

Worksheet 11, Tuesday, December 3, 2013

Critical Numbers

1. Find critical numbers for the function $f(x) = x^{\frac{4}{5}}(x-4)^2$.

2. Find critical numbers for the function $f(x) = x^{\frac{1}{3}} - x^{-\frac{2}{3}}$.

Absolute Extreme Values

3. Find the absolute maximum and absolute minimum values of

$$h(x) = 3(x+1)^{\frac{2}{3}} - x \quad \text{on} \quad [0, 26].$$

4. Find the absolute maximum and absolute minimum values of

$$f(x) = (x-1)^2(2x+10)^2 \quad \text{on} \quad [-3, 2].$$

Related Rates

5. A point moves around the circle $x^2 + y^2 = 9$. When the point is at $(-\sqrt{3}, \sqrt{6})$, its x-coordinate is increasing at the rate of 20 units per second. How fast is its y-coordinate changing at this instant?

6. A 6 foot tall man walks with a speed of 8 feet per second away from a street light that is atop an 18 foot pole. How fast is the top of shadow moving long the ground when he is 100 feet from the light pole?

Limits at Infinity

7. Compute each of the following limits at infinity. Please show your work.

   (a) $\lim_{x\to\infty} \dfrac{x^7 - 4x + 2013}{x^6 + 3x^2 - 8}$

   (b) $\lim_{x\to-\infty} \dfrac{7x^3 - 8x + 19}{4x^3 + 2x^2 - 3x + 5}$

   (c) $\lim_{x\to\infty} \dfrac{1 - x^2}{8x^4 + 200}$

Curve Sketching For the following function, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

8. $f(x) = \dfrac{2x^3 + 45x^2 + 315x + 600}{x^3}$. Take my word for it that (you do NOT have to compute these) $f'(x) = \dfrac{-45(x+4)(x+10)}{x^4}$ and $f''(x) = \dfrac{90(x+5)(x+16)}{x^5}$.
   Hint: You might need the following values:
   $f(-4) = \dfrac{17}{16}$, $f(-10) = \dfrac{1}{20}$, $f(-16) = \dfrac{139}{512}$, and finally $f(-5) = \dfrac{4}{5}$.

Position, Velocity, Acceleration

9. A man stands on the edge of a bridge over a river. He throws a stone straight upward in the air with an initial velocity of 64 feet per second. The ball reaches a height of $\mathbf{s(t) = -16t^2 + 64t + 80}$ feet in t seconds above the water. Answer the following questions:

   (a) What is the intitial height of the stone?
   (b) What is the maximum height the stone reaches?
   (c) What is the stone's velocity at time $t = 1$ second?
   (d) When is the stone 128 feet above the water?
   (e) What is the stone's acceleration at any time t?
   (f) At what time will the stone hit the water?
   (g) What is the stone's velocity when it hits the water?

Turn in your own solutions.