Math 105, Fall 2013

Worksheet 9, Tuesday, November 12, 2013

1. A conical tank, 18 feet across the entire top and 10 feet deep, is leaking water at 9 cubic feet per minute. How fast is the height of the water decreasing when the water level is 1 foot?

   $$\text{**Recall the volume of the cone is given by } V = \frac{1}{3}\pi r^2 h.$$

   NOTE: You should use similar triangles to find a relationship between the radius and the height at general time t.

2. Write out the definition of a **Critical Number** for a function $f(x)$.

3. Find the critical numbers for $f(x) = x\sqrt{1-x}$.

4. Find the critical numbers for $f(x) = \dfrac{x^2+1}{x+3}$

5. Find the critical numbers for $f(x) = x^{\frac{3}{4}} - 2x^{\frac{1}{4}}$

Recall: the **Extreme Value Theorem** states that a continuous function on a bounded interval is guaranteed **both** an absolute maximum and minimum value on that interval.

Question: How do we find those values?

Answer: We use the Closed Interval Method.

Closed Interval Method Supposed that a function f is a continuous function on a closed/bounded interval. To find the absolute maximum and minimum value(s) of f:

Step 1: Find the crticial numbers for f in the interval.

Step 2: Evaluate the function f at the critical numbers from Step 1.

Step 3: Evaluate the function f at the endpoints.

Step 4: Pick off the **largest** of these values from Steps 2 and 3, as the Absolute Maximum Value. Then pick off the **smallest** of these values in Steps 2 and 3, as the Absolute Minimum Value.

6. Find the absolute maximum and absolute minumum value(s) of the function

$$F(x) = x^3 - 3x^2 \text{ on the interval } [-1, 1].$$

7. Find the absolute maximum and absolute minumum value(s) of the function

$$G(x) = (x-1)^2(x-9)^2 \text{ on the interval } [0, 8].$$

8. Find the absolute maximum and absolute minumum value(s) of the function

$$H(x) = \frac{10x}{x^2+1} \text{ on the interval } [0, 2].$$

Turn in your own solutions.