Math 105, Fall 2013

Worksheet 7, Tuesday, October 29, 2013

1. Suppose that f and g are functions, **and**

- $\lim_{x\to 5} f(x) = 6$
- $g(3) = 5$
- $\lim_{x\to 7} g(x) = 1$
- $g(x)$ is continuous at $x = 3$.
- $f(x)$ is continuous at $x = 5$
- $g(7) = 9$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a) $\lim_{x\to 3} g(x) =$

(b) $f(5) =$

(c) Compute $f \circ g(3) =$

(d) Is $g(x)$ continuous at $x = 7$? Why or why not? Use math notation.

2. Compute the derivative of each of the following functions. For these problems, you do **not** need to simplify your derivative.

(a) $y = \dfrac{1}{\left(\dfrac{1}{x^7} + \sqrt{x^6 - 7}\right)^{\frac{6}{7}}}$

(b) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}} + \dfrac{1}{1 + \sqrt{x}} + \dfrac{1}{\sqrt{1 + x}}$

(c) $f(x) = \left(x^4 - \dfrac{4}{x^4}\right)\left(4x + \dfrac{\sqrt{x}}{4}\right)$

(d) $y = \left(\dfrac{1}{3x^8} + 8x^3\right)^{\frac{3}{8}}$

(e) $w(t) = \dfrac{(3t + 1)^4}{(4t + 1)^3}$

3. Find the equation of the tangent line to this curve $y = \dfrac{6x}{\sqrt{x^2 + 3}}$ at the point where $x = 1$.

4. Compute the derivative of $f(x) = \dfrac{6-5x}{2+4x}$ **two** different ways:

   - First use the **limit definition of the derivative**.
   - Second use the Quotient Rule.

   Next, simplify your answer in the first part. Then compute the second derivative $f''(x)$.

5. Find **all** x-coordinates at which the graphs of the following functions have horizontal tangent lines. Please **simplify** your derivatives first. Why?

   (a) $f(x) = (4x+1)^4(7-3x)^8$

   (b) $G(t) = t^3(1-t)^7$

6. Compute the derivative of $f(x) = \dfrac{x}{x-1} + \dfrac{x}{x+1}$. Simplify your answer to a single fraction.

7. Find the equation of the tangent line to the curve $x^3 + x^2y = 6 - 4y^2$ at the point $(1, 1)$.

Turn in your own solutions.