Math 105, Fall 2013

Worksheet 6, Tuesday, October 22, 2013

1. Prove that $y = x^{\frac{2}{3}}$ is not differentiable at $x = 0$.

2. Compute the derivative of each of the following functions. For these problems, you do **not** need to simplify your derivative.

   (a) $y = \sqrt{x}$

   (b) $y = \left(\sqrt{x} + \frac{1}{x}\right)^9$

   (c) $f(x) = \left(x^2 - \frac{5}{x^2}\right)(3x + \sqrt{x})$

   (d) $y = \frac{1}{\sqrt{x^2 - 5x + 3}}$

   (e) $y = \left(\frac{1}{x^3} + 7x\right)^{\frac{5}{7}} \left(x^4 - \frac{1}{x^7}\right)^{-5}$

   (f) $y = \sqrt{\frac{x+5}{5-x}}$

3. Find the equation of the tangent line to this curve $y = \sqrt{x + (x^2+1)^3}$ at the point where $x = 1$.

4. Compute the derivative of $f(x) = \sqrt{x^3 - 4x^2 + \frac{x}{3} - 7}$ **two** different ways.

   - First use the **limit definition of the derivative**.
   - Second use the Chain Rule.

5. Simplify the expression $3(x+1)^2(1-2x)^4 + (x+1)^3 4(1-2x)^3(-2)$

   Hint: Common factors.

6. Solve the equation $\frac{2x+1}{\sqrt{x}} - \frac{x\sqrt{x}}{x+2} = 0$

   Hint: First, put the left hand side over a common denominator. Then remember that a fraction is zero when the numerator is zero.

7. For later purposes we need to practice solving.

   (a) Consider the equation $x^2 + 2xyy' = 3y - 7y'$. Solve for y'.

   (b) Consider the equation $3y^2\frac{dy}{dx} - 5x^3y = 4x + 7\frac{dy}{dx}$. Solve for $\frac{dy}{dx}$.

8. Find **all** x-coordinates at which the graphs of the following functions have horizontal tangent lines. Please **simplify** your derivatives first. Why?

   (a) $f(x) = (7x-3)^4(5x+2)^6$

   (b) $f(x) = (x+1)^2\sqrt{x+2}$

   (c) $w(t) = t^2(1-t)^6$

9. Let $f(x)$ and $g(x)$ be differentiable functions with the following table of values:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-3	2	7
2	-2	6	1	5
3	3	-2	-1	0

   Let

$$h(x) = f(x) \cdot g(x)$$

$$k(x) = \frac{g(x)}{f(x)}$$

$$P(x) = f(x) \cdot f(x)$$

$$Q(x) = f \circ g(x)$$

$$W(x) = g \circ g(x).$$

   Compute $h'(1)$, $k'(3)$, $P'(1)$, $Q'(2)$, and $W'(1)$.

   Note: this problem is testing whether you know your differentiation rules, especially in the case when you don't know the actual function's ($f(x)$ or $g(x)$) formula. To compute the derivative at one specific x-value, you just need the derivative information of each function piece *at* that specific x-value. You don't need to know the entire function's formula. Think about which derivative values are required in each problem. Write out the derivative carefully, and then plug in your specific x-value.

Turn in your own solutions.