Math 105, Fall 2013

Worksheet 5, Tuesday, October 8, 2013

1. State the definition for a function $g(x)$ that is continuous at $x = -7$.

2. Consider the function $f(x)$ that is continuous at $x = 3$. Assume that $f(3) = 4$.

   (a) Write the *definition* for $f(x)$ being continuous at $x = 3$.

   (b) Discuss what you know about $\lim_{x \to 3} f(x) =??$ Why? Be clear and justify with mathematical notation.

3. Suppose that f and g are functions, **and**

   - $\lim_{x \to 3} f(x) = 9$
   - $\lim_{x \to 7} g(x) = -6$
   - $\lim_{x \to 4} f(x) = 7$
   - $g(x)$ is continuous at $x = 7$.
   - $f(x)$ is continuous at $x = 4$.

   Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

   (a) $f(4) =$

   (b) $g(7) =$

   (c) Compute $g \circ f(4) =$

   (d) Does $f(3) = 9$? Why or why not? Use math notation.

4. Suppose that f and g are functions, **and**

   - $\lim_{x \to 7} g(x) = 3$
   - $\lim_{x \to 2} g(x) = 6$
   - $f(3) = 2$
   - $g(x)$ is continuous at $x = 7$ and $x = 2$.
   - $\lim_{x \to 3} f(x) = 5$

   Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

   (a) $g(7) =$

   (b) Compute $g \circ f(3) =$

   (c) Compute $f \circ g(7) =$

   (d) Is $f(x)$ continuous at $x = 3$? Why or why not? Use math notation.

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Definition: The **Derivative of a function** f **at a number** a, denoted by $f'(a)$, is given by

$$(*) \quad \boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

By the definition from class, this value is the slope of the tangent line **at** the given point $(a, f(a))$. This value captures the steepness of the curve **at** that point.

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5. Suppose that $f(x) = 5 - 6x + 4x^2$.
   (a) Compute $f'(1)$ using (*) above. (Here $a = 1$)
   (b) Write the **equation of the tangent line** to the curve $y = f(x)$ at the point where $x = 1$.

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If we replace a by a variable x above, we obtain the **derivative function** $f'(x)$ as

$$(**) \quad \boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

We will call this the *limit definition of the derivative.* Here $f'(x)$ is the function that takes in any value x and spits out the derivative at x. That is, the slope of the tangent line at the point $(x, f(x))$.

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6. For each of the following, find $f'(x)$ using the *limit definition of the derivative* (**).
   (a) $f(x) = x^3$
   (b) $f(x) = x^4$
   (c) $f(x) = \sqrt{x}$
   (d) $f(x) = \dfrac{1}{x}$
   (e) $f(x) = \dfrac{x+1}{x-1}$
   (f) $f(x) = \dfrac{1}{\sqrt{x}}$

Turn in solutions.