Math 105, Fall 2013

Worksheet 4, Tuesday, October 1, 2013

• Please *show* all of your work and *justify* all of your answers.

1. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(**a**) $\lim\limits_{x\to 7} \dfrac{x^2-4x-21}{x^2-3x} =$

(**b**) $\lim\limits_{x\to 4} \dfrac{x^2-3x-4}{|x-4|} =$

(**c**) $\lim\limits_{x\to 4} \dfrac{x^2-2x-8}{x^2-5x+4} =$

(**d**) $\lim\limits_{x\to -5} \dfrac{\frac{1}{1-x}-\frac{1}{6}}{x^2+3x-10} =$

(**e**) $\lim\limits_{x\to 3} \dfrac{x^2-12x+27}{x^2-6x+9} =$

(**f**) $\lim\limits_{x\to 3} \dfrac{x^2-12x+27}{x^2-9} =$

(**g**) $\lim\limits_{x\to 4} \dfrac{x+2}{4-x} =$

(**h**) $\lim\limits_{x\to -4} \dfrac{x+2}{x+4} =$

(**i**) $\lim\limits_{x\to 2} \dfrac{3-\sqrt{x+7}}{x^2-3x+2} =$

(**j**) $\lim\limits_{x\to 1} \dfrac{G(x+2)+x-8}{G(2x)-3x^2-3x+2} =$ where $G(x)=(x-1)^2+3$

(**k**) $\lim\limits_{x\to 7} \dfrac{x-7}{|7-x|} =$

(**l**) $\lim\limits_{x\to 5} \dfrac{f(x^2)-28}{(f(x))^2-10x-14} =$ where $f(x)=x+3$

(**m**) $\lim\limits_{x\to 2} \dfrac{x^2-9x+14}{x^2-4x+4} =$

(**n**) $\lim\limits_{x\to 7} \dfrac{x^2-2x-35}{x^2-2x+1} =$

2.

(a) Suppose that $f(x)=\sqrt{x}$. Compute the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

Simplify until you cancel the h in the denominator.

(b) Suppose that $f(x)=\sqrt{x^2-5x+3}$. Compute the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

Simplify until you cancel the h in the denominator.

(b) Suppose that $f(x)=\dfrac{1-3x}{x+2}$. Compute the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

Simplify until you cancel the h in the denominator.

3. Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3}+1 & \text{if } x > 3 \\ 0 & \text{if } x = 3 \\ (x-2)^2 & \text{if } 1 < x < 3 \\ x + \dfrac{1}{3} & \text{if } 0 < x \leq 1 \\ \dfrac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\lim_{x \to -3} f(x) =$

(d) Compute $\lim_{x \to 0} f(x) =$

(e) Compute $\lim_{x \to 1} f(x) =$

(f) Compute $\lim_{x \to 3} f(x) =$

(g) State all the value(s) at which f is discontinuous. Justify your answer(s) using the definition of continuity.