Math 105 Fall 2013

Worksheet 2, Tuesday, September 17, 2013

1. Let $g(x) = \dfrac{\dfrac{x}{x-1} - \dfrac{x+2}{x}}{x-2}$. Simplify $g(x)$. What is the domain of $g(x)$?

2. Given two functions f and g. The **Composition** of f and g is defined by

$$f \circ g(x) = f(g(x))$$

(a) Discuss what the Domain of $f \circ g$ is.

(b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x + 2$. Compute **and** graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does it mean for two functions to be equal?)

3. Let $f(x) = \dfrac{x+1}{x-1}$. Compute $f(f(2))$. Compute and simplify $f(f(x))$. Hint: first find a large formula for $f(f(x))$. Then simplify by finding common denominators.

4. Let $f(x) = \dfrac{1}{x+1}$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.
warning: $f(x+h) \neq f(x) + h$ **be careful!**

5. Let $f(x) = \dfrac{x-7}{x+3}$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

6. Simplify each of the following expressions.

(a) $\dfrac{x^2 + 6x + 8}{x^2 - 4}$

(b) $\dfrac{x^2 + 6x + 8}{x^2 - 5x - 14}$

(c) $\dfrac{x^2 - 6x + 8}{x^2 - x - 2}$

(d) $\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t}$

(e) $\dfrac{t-1}{g(t^2) - 3}$, where $g(t) = 2t + 1$

(f) $\dfrac{x^2 - 13x + 42}{x^2 - 4x + 12}$

(g) $\dfrac{1}{x} - \dfrac{1}{|x|}$ Hint: you might need two cases here. Write out the definition of $|x|$.

(h) $\dfrac{|x+4|}{x+4}$ Hint: you might need two cases here. Write out the definition of $|x+4|$.

(i) Let $f(x) = \dfrac{1}{x}$. Compute and simplify $\dfrac{f(t-1) - 2f(t)}{t^2 - 4}$

Recall that $\lim_{x \to a^+} f(x) = L$ is the **Right Hand Limit**. We will denote this as **RHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach a *from the right of* a. Also recall that $\lim_{x \to a^-} f(x) = L$ is the **Left Hand Limit**. We will denote this as **LHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach a *from the left of* a.

We write $\lim_{x \to a} f(x) = L$ to represent the full **two-sided limit**. We have the following result.

Theorem: $\boxed{\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^+} f(x) = L \text{ and } \lim_{x \to a^-} f(x) = L.}$

That is, the two-sided limit exists at a if and only if **both** the one-sided limits, from the right and from the left, exist and are equal. If RHL$\neq$ LHL then we say the two-sided limit Does Not Exist or **DNE**.

7. Consider the function defined piece-wise by $f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x + 4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$

   Graph $f(x)$ carefully and find its Domain and Range. Compute $\lim_{x \to 1^+} f(x)$. Compute $\lim_{x \to 1^-} f(x)$. Compute $\lim_{x \to 1} f(x)$, if it exists.

8. Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4 \end{cases}$$

   Graph $f(x)$ carefully and find its Domain and Range. Compute $\lim_{x \to 4} f(x)$, if it exists.

   Hint: You might want to investigate one-sided limits on your own...

Turn in your solutions.