Math 105, D. Benedetto

What you need to know for Exam 1

You should know everything from the beginning of the course up to (and including) Section 1.8. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- Appendices A and B: Review equations of lines, and inequalities. Parallel lines have same slope. Perpendicular lines have opposite reciprocal slopes. Know the difference between *point-slope form*, and *slope-intercept form*. Know how to use *slope-intercept form* to sketch lines quickly and efficiently.

- 1.1: What is a function? (Vertical line test.) Domain and Range of a function. Know how to quickly sketch all the standard functions like x^2, x^3, $\frac{1}{x}$, $\sqrt{x}$, and $|x|$. Piecewise-defined functions, including using all these functions.

- 1.2: Standard functions: polynomials (including constant and linear), power functions (including root functions and things like $\frac{1}{x}\Big)$, rational functions.

- 1.3: Adding, subtracting, multiplying, and dividing functions. Composition of functions. Given the graph of $y = f(x)$, what do $y = f(x) \pm c$, $y = f(x \pm c)$, $y = cf(x)$, $y = f(cx)$, $y = -f(x)$, $y = f(-x)$, and $y = |f(x)|$ look like? (See boxes on pp. 36–37, and Example 5.)

- 1.5: Limits. Intuitive idea of what a limit is. Estimating limits using tables and computations. Be familiar with the various ways a limit can fail to exist. Right- and left-hand limits; and Box 3, page 55. Infinite limits. Vertical asymptotes.

- 1.6: Limit laws: sum, difference, constant multiple, product, quotient, power, root laws. Computing limits of polynomials, rational functions, and other functions. Computing harder limits, like 0/0, using algebraic tricks. What to do in each case of a DSP attempt:

  When you get $\frac{\text{constant, including 0}}{\text{non-zero constant}}$ then it's just DSP.

  When you get $\frac{\text{non-zero constant}}{0}$ then you use *infinite sign analysis*. Study the RHL and LHL cases carefully. Yes it will be an infinite limit, but what sign is attached? Pay attention to your two-point argument: 1. size and 2. sign.

  When you get $\frac{0}{0}$ then you use an algebraic technique to cancel the zero factor. Use factoring, common denominator, or conjugate techniques. It means **MORE WORK!**

  Finally, if you see absolute values, certainly for a $\frac{0}{0}$ case, then you must explore the two RHL and LHL cases.

- 1.8: Continuity: at a number a, or on an interval I. Know both the official definition $\lim_{x \to a} f(x) = f(a)$ and the intuitive idea (you can draw it without lifting your pencil off the paper). The three ways continuity can fail ($f(a)$ not defined; or $\lim_{x \to a} f(x)$ not defined; or both defined, but not equal). Continuity from the right or left.

Some Things You Don't Need to Know For This Exam

- 1.1: Symmetry. Increasing and Decreasing (for right now anyhow).
- 1.2: Algebraic functions. (That is, you don't need to know the definition, though you **do** need to be able to deal with functions like $f(x) = \sqrt{x^2 - 3}$.)
- 1.2: Exponential and logarithmic functions (for right now anyhow).
- 1.6: The identifying numbers of all the limit laws.
- 1.6: The greatest integer function.
- 1.6: Boxes 2 or 3, page 68. limits.
- 1.8: The Intermediate Value Theorem (for right now anyhow).

Tips

- You will be allowed to do the problems in any order, but **make sure you don't accidentally skip any** if you like to jump around.
- Make sure you know how to manipulate functions. If any of the problems from Sections 1.1–1.3 gave you serious trouble, make sure you get things cleared up.
- You will need to compute some limits carefully with full justification. Be ready. Know the algebra tricks you need for $\frac{0}{0}$ limits. Know when to use LHL and RHL sign analysis. Also know how to examine the two cases for absolute value pieces of your functions.
- Make sure you are clear when making a statement about discontinuity. You must tell me which of the three pieces of the definition fail(s).
- Do not write $\lim_{x \to a}$ by itself without the function.
- If you make a declaration about a RHL or LHL limit, then write it out formally using math notation. Look to put forward your *best presentation* for each solution.
- For your piece-wise defined graphs, make sure that when you piece together the puzzle of individual function pieces, that you pay attention to the end-point output values where those cuts are. That will help you determine whether the function pieces line up. Indeed that might change a check of continuity at a given point.
- Justify all of your work!! Don't just use random words to explain answers; I will look for clear mathematical statements.