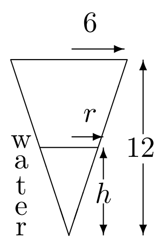
Extra Example for Related Rates

Professor D. Benedetto, Math 105

1. A conical reservoir, 12 ft. deep and also 12 ft. across the top is being filled with water at the rate of 5 cubic feet per minute. How fast is the water rising when it is 4 feet deep?

The cross section (with water level drawn in) looks like:

• Diagram



• Variables

Let r = radius of the water level at time t

Let h = height of the water level at time t

Let V = volume of the water in the tank at time t

Find $\frac{dh}{dt} =?$ when $h = 4$ feet

and $\frac{dV}{dt} = 5\frac{\text{ft}^3}{\text{sec}}$

• Equation relating the variables:

Volume= $V = \frac{1}{3}\pi r^2 h$

• Extra solvable information: Note that r is not mentioned in the problem's info. But there is a relationship, via similar triangles, between r and h. We must have

$$\frac{r}{6} = \frac{h}{12} \implies r = \frac{h}{2}$$

After substituting into our previous equation, we get:

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$ Now the equation is in terms of just V and h.

• Differentiate both sides w.r.t. time t.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{1}{12}\pi h^3\right) \implies \frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt} \implies \frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} \text{ (Related Rates!)}$$

• Substitute Key Moment Information (now and not before now!!!):

$$5 = \frac{1}{4}\pi(4)^2\frac{dh}{dt}$$

• Solve for the desired quantity:

$$\frac{dh}{dt} = \frac{5 \cdot 4}{16\pi} = \frac{5}{4\pi}\frac{\text{ft}}{\text{sec}}$$

• Answer the question: The water is rising at a rate of $\frac{5}{4\pi}$ feet every second at that moment.

(Notice this problem the tank is filling, if it were leaking, just change the signs on the change of volume information.)