Math 105 Quiz #7 November 11, 2013 Answer Key

• This is a closed-book quiz. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

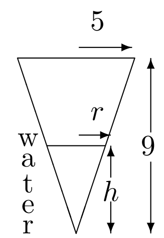
• Please *show* all of your work and *justify* all of your answers.

1. [10 Points] A conical tank, 10 feet across the entire top and 9 feet deep, is leaking water. The radius of the water level is decreasing at the rate of 2 feet per minute. How fast is the water leaking out of the tank when the radius of the water level is 1 foot?

$$\text{**Recall the volume of the cone is given by } V = \frac{1}{3}\pi r^2 h.$$

The cross section (with water level drawn in) looks like:

• Diagram



• Variables
Let $r =$ radius of the water level at time t
Let $h =$ height of the water level at time t
Let $V =$ volume of the water in the tank at time t
Find $\dfrac{dV}{dt} =$? when $r = 1$ foot
and $\dfrac{dr}{dt} = -2\dfrac{\text{ft}}{\text{min}}$

• Equation relating the variables:

Volume= $V = \dfrac{1}{3}\pi r^2 h$

• Extra solvable information: Note that h is not mentioned in the problem's info. But there is a relationship, via similar triangles, between r and h. We must have

$$\frac{r}{5} = \frac{h}{9} \implies h = \frac{9r}{5}$$

After substituting into our previous equation, we get:

$$V = \frac{1}{3}\pi r^2 \left(\frac{9r}{5}\right) = \frac{3}{5}\pi r^3$$

• Differentiate both sides w.r.t. time t.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{3}{5}\pi r^3\right) \implies \frac{dV}{dt} = \frac{9}{5}\pi r^2 \cdot \frac{dr}{dt} \text{ (Related Rates!)}$$

• Substitute Key Moment Information (now and not before now!!!):

$\frac{dV}{dt} = \frac{9}{5}\pi r^2 \cdot \frac{dr}{dt} = \frac{9}{5}\pi(1)^2(-2)$

• Solve for the desired quantity:

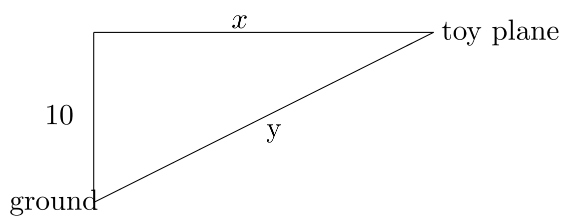
$\frac{dV}{dt} = -\frac{18\pi}{5}\frac{\text{ft}^3}{\text{min}}$

• Answer the question that was asked: The water is *leaking out* at a rate of $\frac{18\pi}{5}$ cubic feet every minute at that moment.

2. [10 Points] A boy is lying on the ground with a remote control airplane. At time $t = 0$ seconds, the toy airplane starts 10 feet above the boy on the ground. It is flying horizontally at a rate of 5 feet per second. At what rate is the distance between the airplane and the boy on the ground increasing when 4 seconds has passed?

• Diagram

The picture at arbitrary time t is:



• Variables

Let x = distance toy plane has travelled horizontally at time t

Let y = distance between plane and boy on ground at time t

Find $\frac{dy}{dt}$ =? when $t = 4$ sec.

and $\frac{dx}{dt} = 5\frac{\text{ft}}{\text{sec}}$

• Equation relating the variables:

We have $x^2 + (10)^2 = y^2$ by the Pythagorean Theorem.

$x^2 + 100 = y^2$

• Differentiate both sides w.r.t. time t.

$\frac{d}{dt}(x^2 + 100) = \frac{d}{dt}(y^2) \implies 2x\frac{dx}{dt} = 2y\frac{dy}{dt} \implies x\frac{dx}{dt} = y\frac{dy}{dt}$ (Related Rates!)

• Extra solvable information:

At the key instant when $t = 4$ seconds, using the original rate of the horizontal travel of the kite, we have

Distance =Rate times Time.

So at that key moment, 4 seconds later, $x = \frac{dx}{dt}(t) = (5)(4) = 20$ feet.

We also need to find the distance y at that key moment:

We can use the Pythagorean Theorem again

$y = \sqrt{(20)^2 + ((10)^2} = \sqrt{400 + 100} = \sqrt{500} = 10\sqrt{5}$

• Substitute Key Moment Information (now and not before now!!!):

$(20)(5) = 10\sqrt{5}\frac{dy}{dt}$

• Solve for the desired quantity:

$\frac{dy}{dt} = \frac{100}{10\sqrt{5}} = \frac{10}{\sqrt{5}}\frac{\text{ft}}{\text{sec}}$

• Answer the question that was asked: The distance between the boy and the toy plane is *increasing* at a rate of $\frac{10}{\sqrt{5}}$ feet per second at that moment.