ANSWER KEY

Math 105 Quiz #6 October 28, 2013

1. [5 Points] Differentiate the following functions. Please do **NOT** simplify your derivatives here.

1. $f(x) = 5x^2 - \sqrt{x} + x^{\frac{5}{7}} + \dfrac{3}{x} - \dfrac{2}{x^2}$

   $f'(x) = \boxed{10x - \dfrac{1}{2\sqrt{x}} + \dfrac{5}{7x^{\frac{2}{7}}} - \dfrac{3}{x^2} + \dfrac{4}{x^3}}$

2. $f(x) = \left(x^5 + \sqrt{x}\ \right)\left(\dfrac{1}{x} - 3x^2\right)$

   Two ways to solve for this derivative:

   First simplify $f(x)$ and then differentiate.

   $f(x) = x^4 - 3x^7 + \dfrac{1}{\sqrt{x}} - 3x^{\frac{5}{2}}$

   $f'(x) = \boxed{4x^3 - 21x^6 - \dfrac{1}{2x^{\frac{3}{2}}} - \dfrac{15}{2}x^{\frac{3}{2}}}$

   OR Second, differentiate with the product rule (and then check they are equal by simplifying, although that was not asked for here).

   $f'(x) = \boxed{\left(x^5 + \sqrt{x}\ \right)\left(-\dfrac{1}{x^2} - 6x\right) + \left(\dfrac{1}{x} - 3x^2\right)\left(5x^4 + \dfrac{1}{2\sqrt{x}}\ \right)}$

   It's not required here, but if you simplify you see you get the same answer as above.

   $f'(x) = -x^3 - 6x^6 - \dfrac{1}{x^{\frac{3}{2}}} - 6x^{\frac{3}{2}} + 5x^3 + + \dfrac{1}{2x^{\frac{3}{2}}} - 15x^6 - \dfrac{3}{2}x^{\frac{3}{2}}$

   $= 4x^3 - 21x^6 - \dfrac{1}{2x^{\frac{3}{2}}} - \dfrac{15}{2}x^{\frac{3}{2}}$

3. $f(x) = \dfrac{\sqrt{x} - \dfrac{1}{x^8}}{x^7}$

   Two ways to solve for this derivative:

   First simplify $f(x)$ and then differentiate.

   $f(x) = \dfrac{1}{x^{\frac{13}{2}}} - \dfrac{1}{x^{15}}$

   $f'(x) = \boxed{-\dfrac{13}{2x^{\frac{15}{2}}} + \dfrac{15}{x^{16}}}$

   OR Second, differentiate with the quotient rule (and then check they are equal by simplifying, although that was not asked for here).

$$f'(x) = \boxed{\frac{x^7\left(\frac{1}{2\sqrt{x}} + \frac{8}{x^9}\right) - \left(\sqrt{x} - \frac{1}{x^8}\right)(7x^6)}{x^{14}}}$$

It's not required here, but if you simplify you see you get the same answer as above.

$$f'(x) = \frac{\frac{1}{2}x^{\frac{13}{2}} + \frac{8}{x^2} - 7x^{\frac{13}{2}} + \frac{7}{x^2}}{x^{14}}$$

$$= \frac{-\frac{13}{2}x^{\frac{13}{2}} + \frac{15}{x^2}}{x^{14}} = -\frac{13}{2x^{\frac{15}{2}}} + \frac{15}{x^{16}}$$

4. $f(x) = \sqrt{8x^4 + \sqrt{x^4+8}}$

$$f'(x) = \frac{1}{2\sqrt{8x^4 + \sqrt{x^4+8}}}\left(32x^3 + \frac{1}{2\sqrt{x^4+8}}(4x^3)\right)$$

5. $f(x) = \left(3x + \left(x^2+7x\right)^4\right)^{\frac{2}{3}}$

$$f'(x) = \boxed{\frac{2}{3}\left(3x + \left(x^2+7x\right)^4\right)^{-\frac{1}{3}}\left(3 + 4\left(x^2+7x\right)^3(2x+7)\right)}$$

2. [5 Points] Compute the equation of the line that is tangent to the curve $f(x) = \dfrac{1}{x-7}$ at the point where $x = 1$. **Also** draw the graph of the given function **and** the graph of this specific tangent line all on one skech.

Here $f'(x) = -\dfrac{1}{(x-7)^2}$, so the specific slope if given by

$$f'(1) = -\frac{1}{(1-7)^2} = -\frac{1}{36}$$

The equation of the tangent line through the point $(1, f(1)) = \left(1, -\dfrac{1}{6}\right)$ with slope $f'(1) = -\dfrac{1}{36}$ is given by point-slope form as

$$y - \left(-\frac{1}{6}\right) = -\frac{1}{36}(x-1)$$

or $y + \dfrac{1}{6} = -\dfrac{1}{36}x + \dfrac{1}{36}$

or $y = -\dfrac{1}{36}x + \dfrac{1}{36} - \dfrac{1}{6}$

or $y = -\dfrac{1}{36}x + \dfrac{1}{36} - \dfrac{6}{36}$

or $\boxed{y = -\dfrac{1}{36}x - \dfrac{5}{36}}$

3. [10 Points] Consider the function

$$f(x) = (3x-2)^5(5x+1)^6.$$

Find **all** x-coordinates at which the graph of this function has horizontal tangent lines. Please **simplify** your derivatives by factoring out common factors.

First compute the derivative:

$$\begin{aligned} f'(x) &= (3x-2)^5 6(5x+1)^5(5) + (5x+1)^6 5(3x-2)^4(3) \\ &= 15(3x-2)^4(5x+1)^5\,[2(3x-2)+(5x+1)] \\ &= 15(3x-2)^4(5x+1)^5\,[6x-4+5x+1] \\ &= 15(3x-2)^4(5x+1)^5\,[11x-3] \overset{\text{set}}{=} 0 \end{aligned}$$

Here either $3x-2=0$ or $5x+1=0$ or $11x-3=0$.

Which implies $\boxed{x=\frac{2}{3}}$ or $\boxed{x=-\frac{1}{5}}$ or $\boxed{x=\frac{3}{11}}$