ANSWER KEY

Math 105 Quiz #5 October 21, 2013

1. [5 Points] Prove that the function $f(x) = |x-7|$ is **not** differentiable at $x = 7$.

We need to show that $f'(7)$ does not exist.

$$f'(7) = \lim_{h\to 0} \frac{f(7+h)-f(7)}{h} = \lim_{h\to 0} \frac{|7+h-7|-|7-7|}{h} = \lim_{h\to 0} \frac{|h|-0}{h} = \lim_{h\to 0} \frac{|h|}{h}$$

$\boxed{\text{Does Not Exist b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$

LHL: $\lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$

Recall $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

2. [5 Points] Compute the equation of the line that is tangent to the curve $y = (3x^2+5)(2-4x)$ at the point where $x = 1$.

To compute the derivative we can use algebra first followed by some power rules.

$y = (3x^2+5)(2-4x) = 6x^2 - 12x^3 + 10 - 20x = -12x^3 + 6x^2 - 20x + 10$

$y'(x) = -36x^2 + 12x - 20$

OR we could use the Product Rule, followed by some algebra.

$y'(x) = (3x^2+5)(-4) + (2-4x)(6x) = -12x^2 - 20 + 12x - 24x^2 = -36x^2 + 12x - 20$

The slope at $x = 1$ is given by $y'(1) = -36 + 12 - 20 = -42$

Note that $y(1) = (3+5)(2-4) = (8)(-2) = -16$

The point is $(1, y(1)) = (1, -16)$

The equation of the tangent line is given by

$y - (-16) = -42(x-1)$

or

$y + 16 = -42x + 42$

or

finally $\boxed{y = -42x + 26}$

3. [10 Points] Consider the function $f(x) = \dfrac{7x+3}{1-5x}$. Compute the derivative $f'(x)$ in two different ways:

(a) First compute the derivative using the **limit definition of the derivative**.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{7(x+h)+3}{1-5(x+h)} - \dfrac{7x+3}{1-5x}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{[7x+7h+3](1-5x)-(7x+3)[1-5x-5h]}{(1-5(x+h))(1-5x)}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\dfrac{7x+7h+3-35x^2-35xh-15x-7x+35x^2+35xh-3+15x+15h}{(1-5(x+h))(1-5x)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\dfrac{22h}{(1-5(x+h))(1-5x)}\right)}{h} = \lim_{h\to 0} \frac{22h}{(1-5(x+h))(1-5x)}\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{22}{(1-5(x+h))(1-5x)} = \boxed{\frac{22}{(1-5x)^2}}$$

(b) Second compute the derivative using the **Quotient Rule**.

$$f'(x) = \frac{(1-5x)(7)-(7x+3)(-5)}{(1-5x)^2} = \frac{7-35x+35x+15}{(1-5x)^2} = \boxed{\frac{22}{(1-5x)^2}}$$