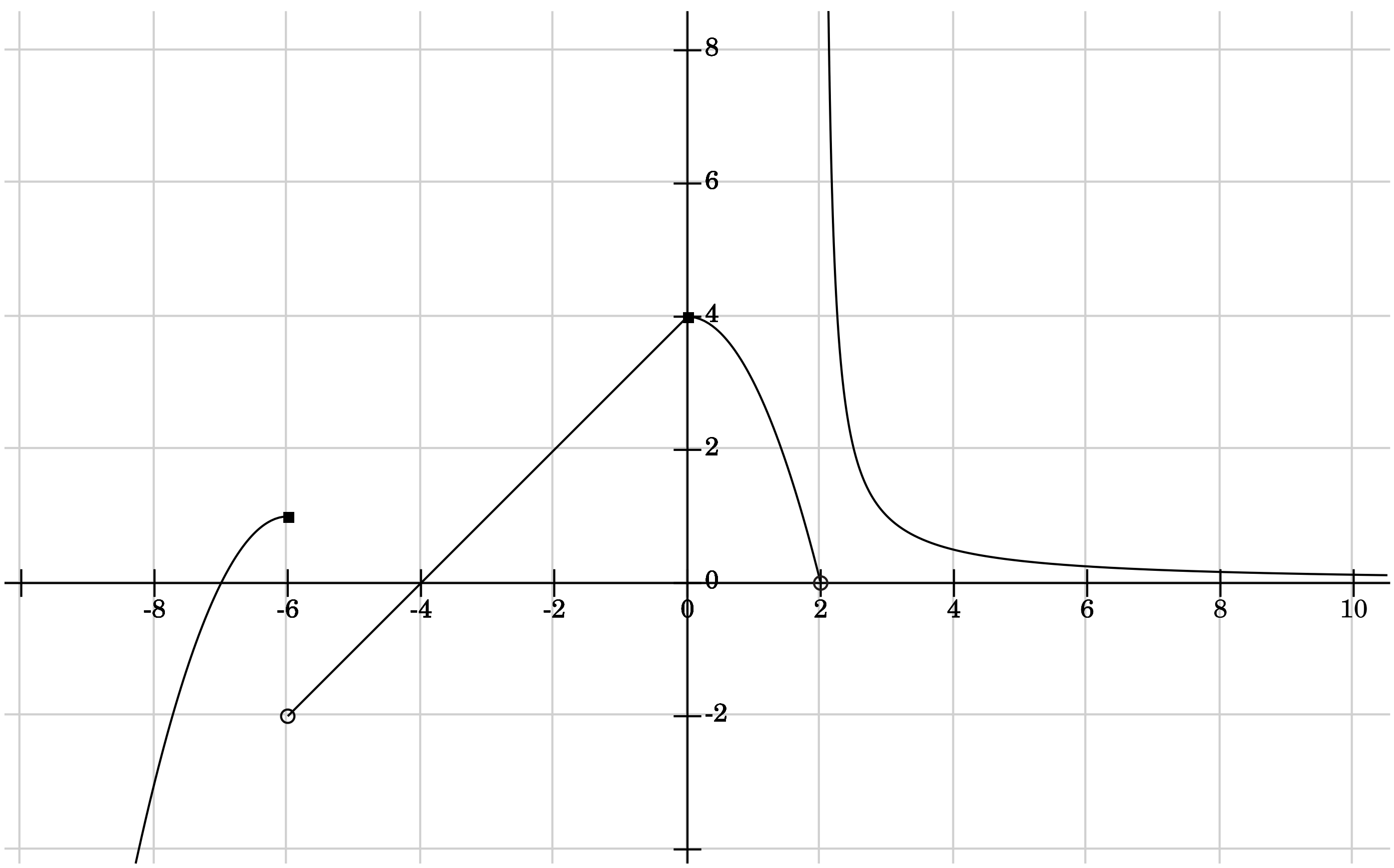
ANSWER KEY

Math 105 Quiz #4 September 30, 2013

1. [10 Points] Consider the function defined by

$$f(x) = \begin{cases} \dfrac{1}{x-2} & \text{if } \ x > 2 \\ 4 - x^2 & \text{if } \ 0 \le x < 2 \\ x + 4 & \text{if } \ -6 < x < 0 \\ 1 - (x+6)^2 & \text{if } \ x \le -6 \end{cases}$$

Graph $f(x)$.



Answer the following questions. Justify your answers.

(a) $\lim\limits_{x\to-6} f(x) = \boxed{\text{DNE b/c RHL}\neq\text{LHL}}$

RHL: $\lim\limits_{x\to-6^+} f(x) = -2$

LHL: $\lim\limits_{x\to-6^-} f(x) = 1$

(b) $\lim\limits_{x\to 0} f(x) = \boxed{4}$ b/c RHL=LHL

RHL: $\lim\limits_{x\to 0^+} f(x) = 4$

LHL: $\lim\limits_{x\to 0^-} f(x) = 4$

(c) $\lim\limits_{x\to 2} f(x) = \boxed{\text{DNE b/c RHL}\neq\text{LHL}}$

RHL: $\lim\limits_{x\to 2^+} f(x) = +\infty$

LHL: $\lim\limits_{x\to 2^-} f(x) = 0$

2. [10 Points] Compute each of the following limits. Justify your answers. Be clear if the limit equals a value, $+\infty$, $-\infty$, or Does Not Exist.

(a) $\lim\limits_{x\to 7} \dfrac{3-x}{x-7}$ $\boxed{\text{DNE b/c RHL}\neq\text{LHL}}$

RHL: $\lim\limits_{x\to 7^+} \dfrac{3-x}{x-7} = \dfrac{-4}{0^+} = -\infty$

LHL: $\lim\limits_{x\to 7^-} \dfrac{3-x}{x-7} = \dfrac{-4}{0^-} = +\infty$

(b) $\lim\limits_{x\to -7} \dfrac{x+7}{x^2+x+1} \overset{\text{DSP}}{=} \dfrac{0}{43} = \boxed{0}$

(c) $\lim\limits_{x\to -7} \dfrac{x+7}{x^2+2x-35} \overset{\frac{0}{0}}{=} \lim\limits_{x\to -7} \dfrac{x+7}{(x+7)(x-5)} = \lim\limits_{x\to -7} \dfrac{1}{x-5} = \boxed{-\dfrac{1}{12}}$

(d) $\lim\limits_{x\to 2} \dfrac{\sqrt{x+7}-3}{x^2-3x+2} = \lim\limits_{x\to 2} \dfrac{\sqrt{x+7}-3}{x^2-3x+2} \cdot \dfrac{\sqrt{x+7}+3}{\sqrt{x+7}+3} = \lim\limits_{x\to 2} \dfrac{(x+7)-9}{(x^2-3x+2)(\sqrt{x+7}+3)}$

$= \lim\limits_{x\to 2} \dfrac{x-2}{(x-2)(x-1)(\sqrt{x+7}+3)} = \lim\limits_{x\to 2} \dfrac{1}{(x-1)(\sqrt{x+7}+3)} \overset{\text{L.L.}}{=} \dfrac{1}{\sqrt{9}+3)} = \dfrac{1}{\sqrt{9}+3} = \boxed{\dfrac{1}{6}}$

(e) $\lim\limits_{x\to -7} \dfrac{\dfrac{1}{1-x} - \dfrac{1}{8}}{x+7} \overset{\frac{0}{0}}{=} \lim\limits_{x\to -7} \dfrac{\dfrac{8-(1-x)}{(1-x)(8)}}{x+7} = \lim\limits_{x\to -7} \dfrac{\dfrac{7+x}{(1-x)(8)}}{x+7}$

$$= \lim_{x\to -7} \frac{7+x}{(1-x)(8)} \cdot \frac{1}{x+7} = \lim_{x\to -7} \frac{1}{(1-x)(8)} = \frac{1}{(1-(-7))(8)} = \frac{1}{(8)(8)} = \boxed{\frac{1}{64}}$$

(f) $\lim_{x\to 7} \frac{x-7}{|x-7|}$ $\boxed{\text{DNE b/c LHL} \neq \text{RHL}}$

Recall $|x-7| = \begin{cases} x-7 & x-7 \geq 0 \\ -(x-7) & x-7 < 0 \end{cases} = \begin{cases} x-7 & x \geq 7 & \longleftarrow \text{RHL} \\ -(x-7) & x < 7 & \longleftarrow \text{LHL} \end{cases}$

RHL: $\lim_{x\to 7^+} \frac{x-7}{|x-7|} = \lim_{x\to 7^+} \frac{x-7}{x-7} = 1$

LHL: $\lim_{x\to 7^-} \frac{x-7}{|x-7|} = \lim_{x\to 7^-} \frac{x-7}{-(x-7)} = -1$