ANSWER KEY

Math 105 Quiz #3 September 23, 2013

1. [5 Points] Let $f(x) = \dfrac{1}{x^2}$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

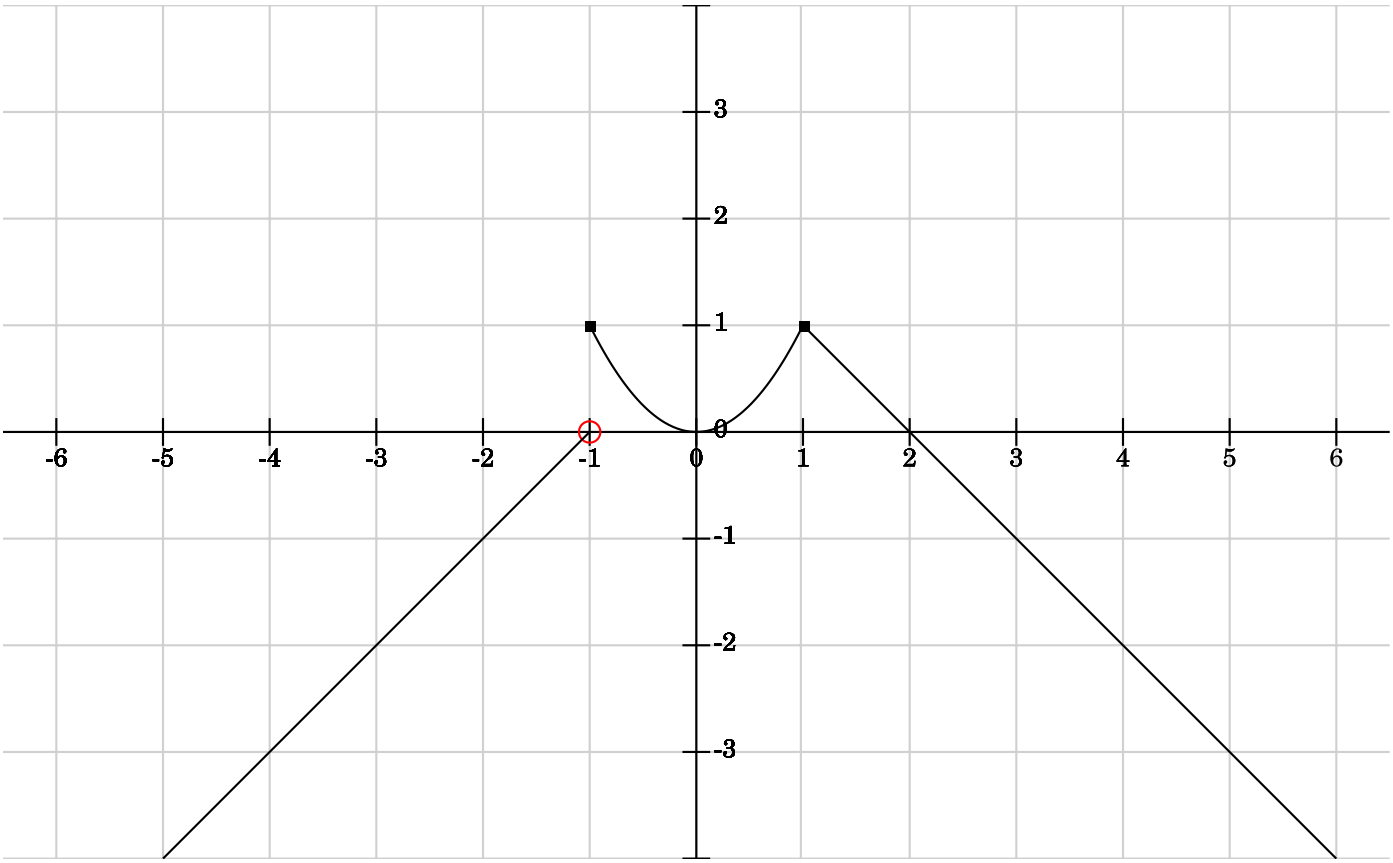
$$\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \frac{\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}}{h} = \frac{\dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}}{h}$$

$$= \frac{\dfrac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}}{h} = \frac{\dfrac{-2xh-h^2}{x^2(x+h)^2}}{h} = \frac{-2xh-h^2}{x^2(x+h)^2} \cdot \left(\frac{1}{h}\right) = \frac{h(-2x-h)}{x^2(x+h)^2} \cdot \left(\frac{1}{h}\right)$$

$$= \boxed{\frac{-2x-h}{x^2(x+h)^2}}$$

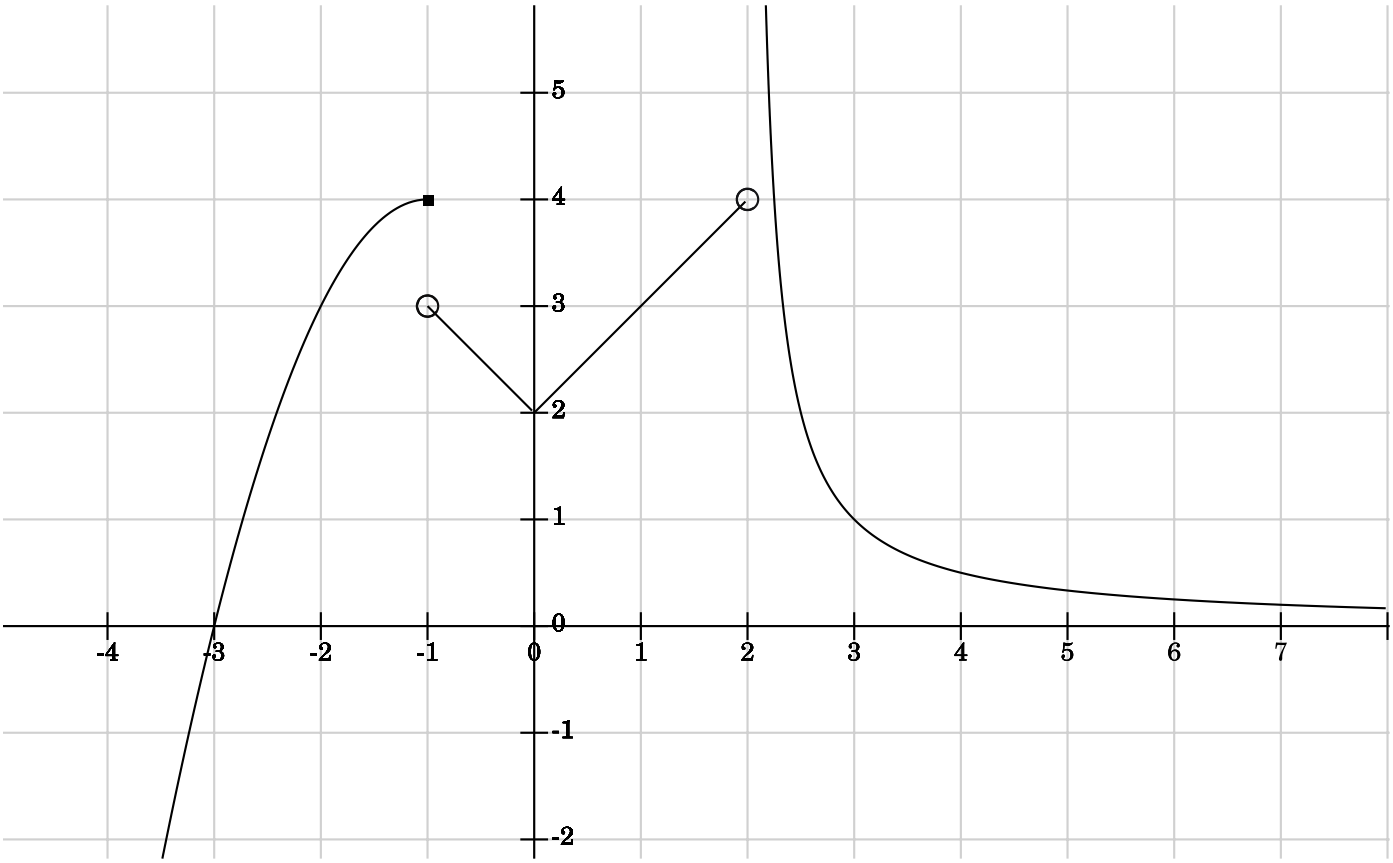
2. [5 Points] Consider the function defined piece-wise by $f(x) = \begin{cases} x+1 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$

Graph $f(x)$. **Then** use it to determine the values a for which $\lim\limits_{x \to a} f(x)$ exists.



The limit exists for all a except $a = -1$.

3. [5 Points] Consider the following graph of $f(x)$. Answer the questions below. Justify your answers when necessary.



(a) $\lim_{x \to 2^+} f(x) = \underline{+\infty}$

(b) $\lim_{x \to 2^-} f(x) = \underline{4}$

(c) $\lim_{x \to 2} f(x) = \underline{\text{DNE b/c RHL } \neq \text{ LHL}}$ See (a) and (b) above

(d) $\lim_{x \to -1} f(x) = \underline{\text{DNE b/c RHL } \neq \text{ LHL}}$

RHL: $\lim_{x \to -1^+} f(x) = 3$

LHL: $\lim_{x \to -1^-} f(x) = 4$

(e) $\lim_{x \to 0} f(x) = \underline{2 \text{ b/c RHL } = \text{ LHL}}$

RHL: $\lim_{x\to 0^+} f(x) = 2$

LHL: $\lim_{x\to 0^-} f(x) = 2$

4. [5 Points] Compute each of the following limits. Justify your answers. Be clear if the limit equals $+\infty$, $-\infty$, or Does Not Exist.

(a) $\lim_{x\to 7} \dfrac{x-2}{x-7}$ $\boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to 7^+} \dfrac{x-2}{x-7} = \dfrac{5}{0^+} = +\infty$

LHL: $\lim_{x\to 7^-} \dfrac{x-2}{x-7} = \dfrac{5}{0^-} = -\infty$

(b) $\lim_{x\to 3} \dfrac{x-5}{x-3}$ $\boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to 3^+} \dfrac{x-5}{x-3} = \dfrac{-2}{0^+} = -\infty$

LHL: $\lim_{x\to 3^-} \dfrac{x-5}{x-3} = \dfrac{-2}{0^-} = +\infty$

(c) $\lim_{x\to -2} \dfrac{x-5}{x+2}$ $\boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to -2^+} \dfrac{x-5}{x+2} = \dfrac{-7}{0^+} = -\infty$

LHL: $\lim_{x\to -2^-} \dfrac{x-5}{x+2} = \dfrac{-7}{0^-} = +\infty$