ANSWER KEY

1. [5 Points]   Consider the function $f(x) = 1 + \dfrac{1}{x}$. Express $f(f(x))$ as a single fraction.

$$f(f(x)) = f\left(1 + \frac{1}{x}\right) = 1 + \frac{1}{1 + \dfrac{1}{x}} = 1 + \frac{1}{\dfrac{x}{x} + \dfrac{1}{x}} = 1 + \frac{1}{\dfrac{x+1}{x}}$$

$$= 1 + \frac{x}{x+1} = \frac{x+1}{x+1} + \frac{x}{x+1} = \frac{x+1+x}{x+1} = \boxed{\frac{2x+1}{x+1}}$$
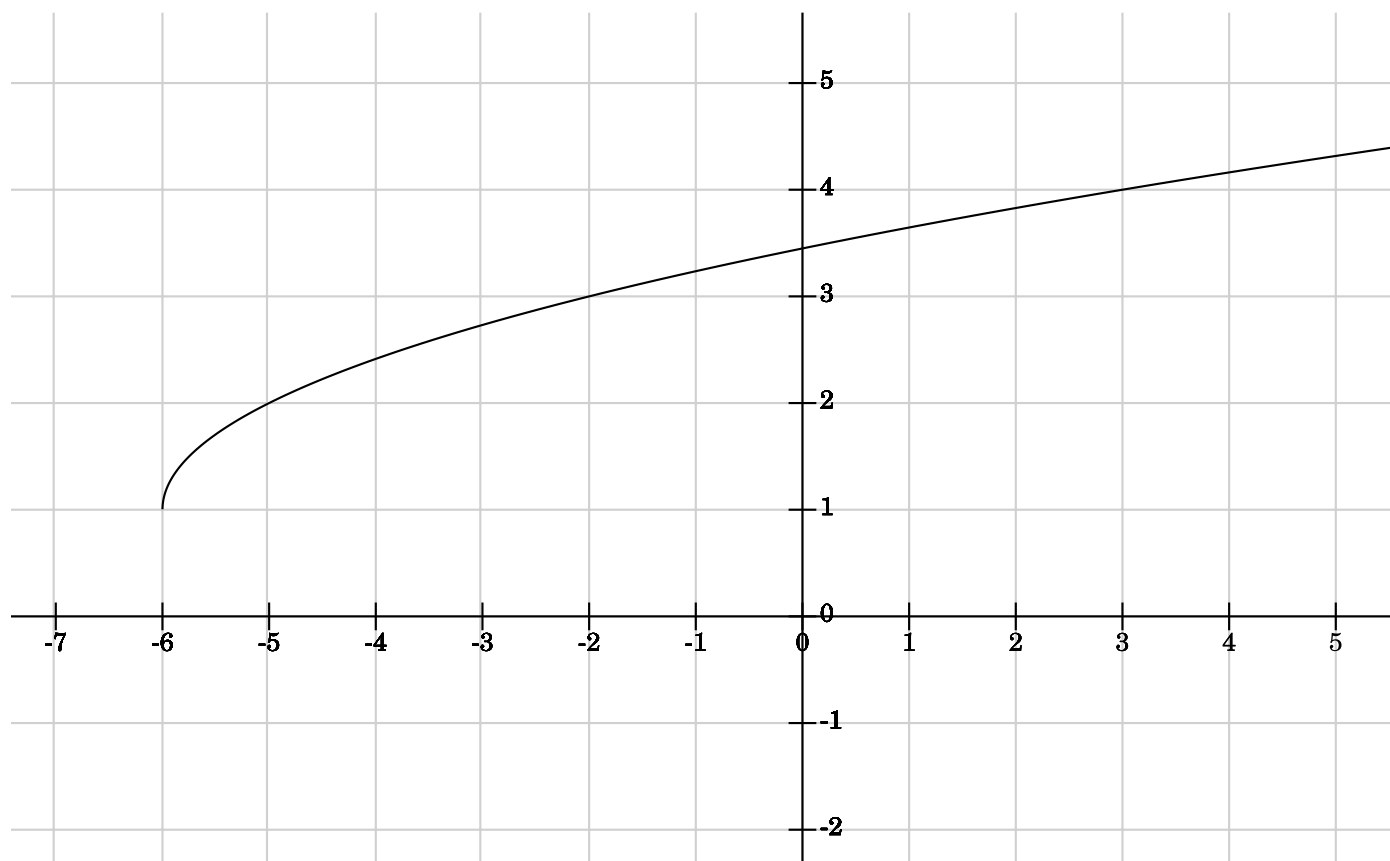
Watch the algebra carefully to find all of the common denominators.

2. [5 Points] Let $f(x) = x + 1$. Let $g(x) = \sqrt{x+6}$.

(a) Compute $f \circ g(x)$. **State** the domain for $f \circ g$. Then **sketch** the graph for $f \circ g$.
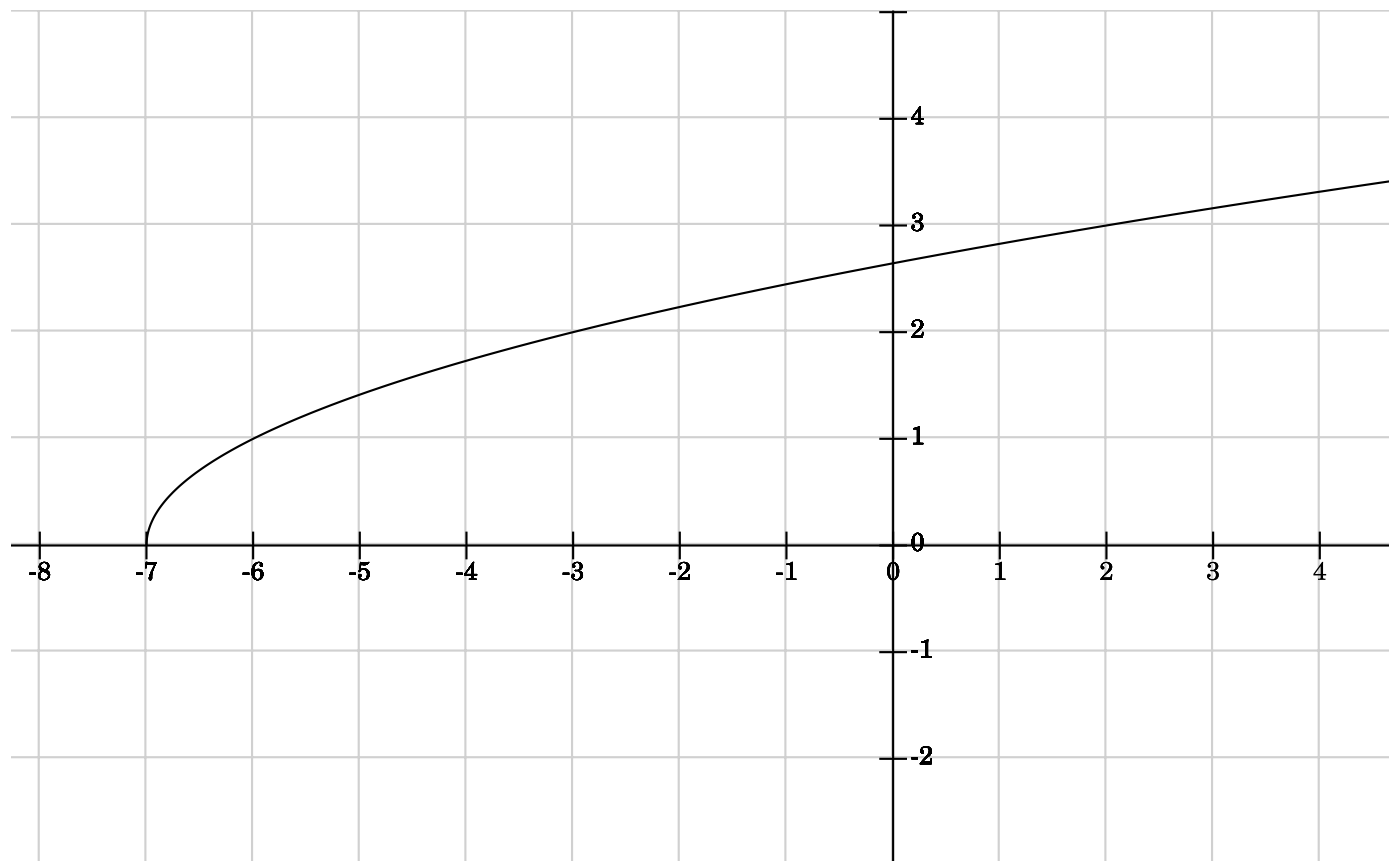
$f \circ g(x) = f(g(x)) = f(\sqrt{x+6}) = \sqrt{x+6} + 1$

Domain=$\{x | x \geq -6\}$.

(b) Compute $g \circ f(x)$. **State** the domain for $g \circ f$. Then **sketch** the graph for $g \circ f$.

$g \circ f(x) = g(f(x)) = g(x+1) = \sqrt{(x+1)+6} = \sqrt{x+7}$
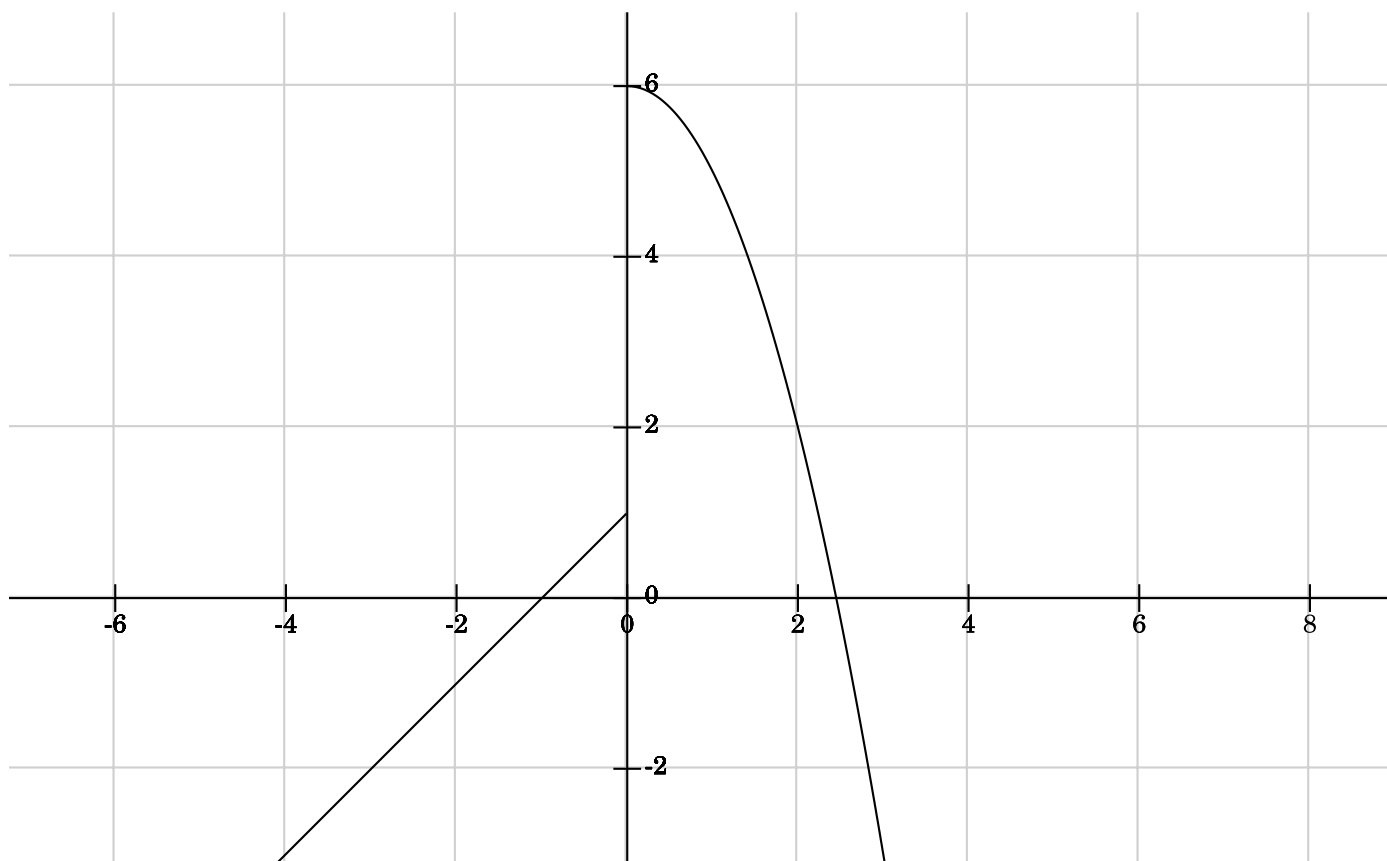
Domain=$\{x | x \geq -7\}$.

(c) Does $f \circ g = g \circ f$? Explain your answer.

No, they are not equal functions since they have different domains. In particular, they do not take the same value at every possible input. Specifically, $f \circ g(1) = f(g(1)) = f(\sqrt{7}) = \sqrt{7} + 1$, whereas $g \circ f(1) = g(f(1)) = g(2) = \sqrt{8}$ They also have different graphs.

3. [5 Points] Consider the function defined piece-wise by $f(x) = \begin{cases} x + 1 & \text{if } x \leq 0 \\ -x^2 + 6 & \text{if } x > 0 \end{cases}$
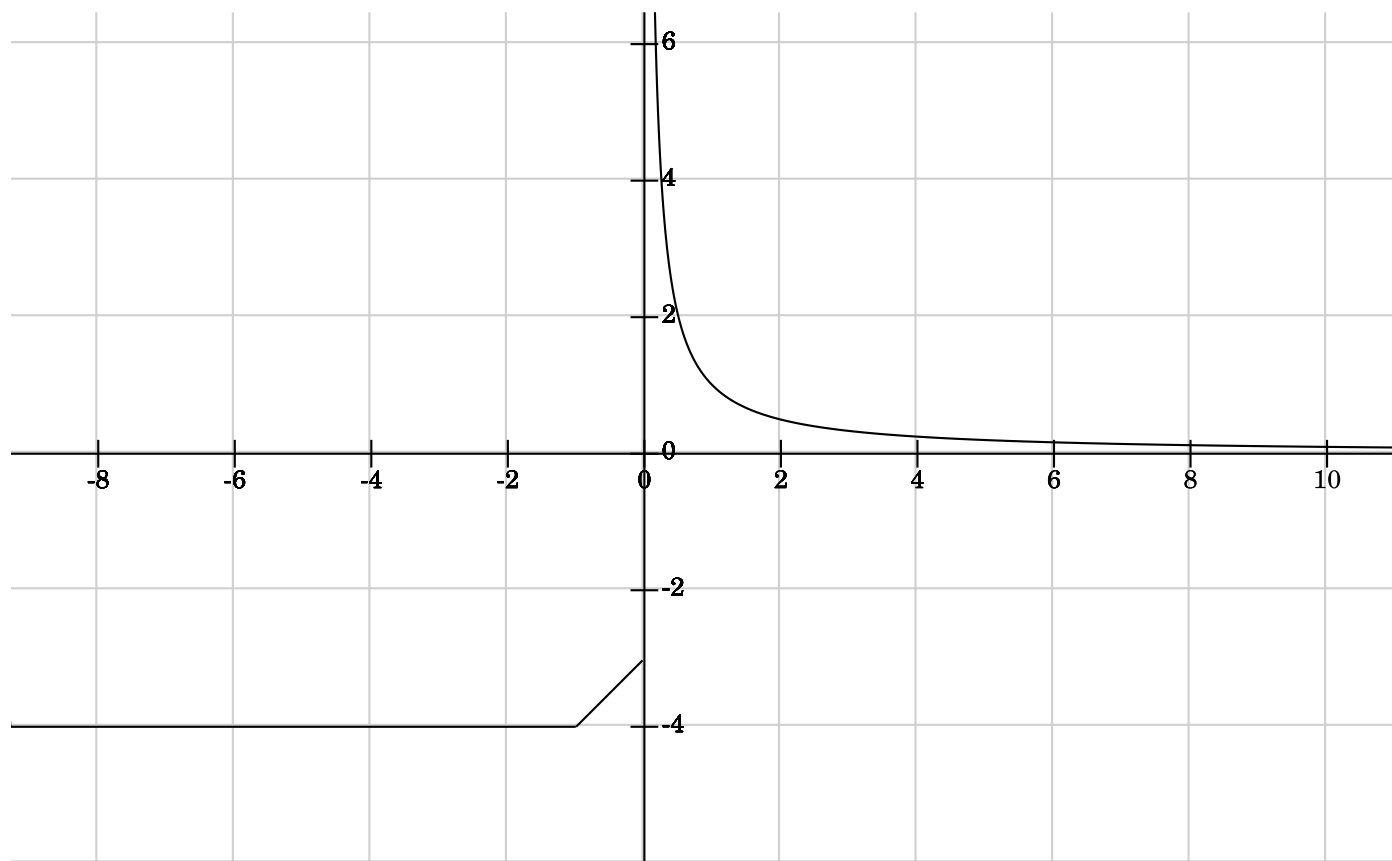
Graph $f(x)$ and state its Domain and Range.

Domain=$\mathbb{R}$.

Range=$\{y|y \le 6\}$.

4. [5 Points] Consider the function defined piece-wise by $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x > 0 \\ x - 3 & \text{if } -1 \le x < 0 \\ -4 & \text{if } x < -1 \end{cases}$

Graph $f(x)$ and state its Domain and Range.

Domain=$\{x|x \neq 0\}$.

Range=$[-4, -3) \cup (0, \infty)$.