Extra Limit Homework 8 Math 105 Due **Friday September 27**

ANSWER KEY

1. $\lim\limits_{\mathbf{x}\to\mathbf{2}} \dfrac{\mathbf{x+7}}{\mathbf{x^2-3x+2}}$ $\boxed{\text{DNE (Does Not Exist) since RHL}\neq\text{LHL}}$

- RHL: $\lim\limits_{x\to 2^+} \dfrac{x+7}{x^2-3x+2} = \lim\limits_{x\to 2^+} \dfrac{x+7}{(x-2)(x-1)} = \dfrac{9}{(0^+)\cdot(1)} = +\infty$
- LHL: $\lim\limits_{x\to 2^-} \dfrac{x+7}{x^2-3x+2} = \lim\limits_{x\to 2^-} \dfrac{x+7}{(x-2)(x-1)} = \dfrac{9}{(0^-)\cdot(1)} = -\infty$

2. $\lim\limits_{\mathbf{x}\to\mathbf{1}} \dfrac{\mathbf{x+7}}{\mathbf{x^2-3x+2}}$ $\boxed{\text{DNE since RHL}\neq\text{LHL}}$

- RHL: $\lim\limits_{x\to 1^+} \dfrac{x+7}{x^2-3x+2} = \lim\limits_{x\to 1^+} \dfrac{x+7}{(x-2)(x-1)} = \dfrac{8}{(-1)\cdot(0^+)} = -\infty$
- LHL: $\lim\limits_{x\to 1^-} \dfrac{x+7}{x^2-3x+2} = \lim\limits_{x\to 1^-} \dfrac{x+7}{(x-2)(x-1)} = \dfrac{8}{(-1)\cdot(0^-)} = +\infty$

3. $\lim\limits_{\mathbf{x}\to\mathbf{0}} \dfrac{\mathbf{x+7}}{\mathbf{x^2-3x+2}} \overset{\text{DSP}}{=} \boxed{\dfrac{7}{2}}$ by Direct Substitution Property (D.S.P.)

4. $\lim\limits_{\mathbf{x}\to\mathbf{-7}} \dfrac{\mathbf{x+7}}{\mathbf{x^2+x+1}} \overset{\text{DSP}}{=} \dfrac{0}{49-7+1} = \dfrac{0}{43} = \boxed{0}$ trust this answer!

5. $\lim\limits_{\mathbf{x}\to\mathbf{-7}} \dfrac{\mathbf{x+7}}{\mathbf{x^2+2x-35}} = \lim\limits_{x\to -7} \dfrac{x+7}{(x+7)(x-5)} = \lim\limits_{x\to -7} \dfrac{1}{x-5} \overset{\text{DSP}}{=} \dfrac{1}{-7-5} = \dfrac{1}{-12} = \boxed{-\dfrac{1}{12}}$

6. $\lim\limits_{\mathbf{x}\to\mathbf{7}} \dfrac{\mathbf{x+7}}{\mathbf{x-7}}$ $\boxed{\text{DNE since RHL}\neq\text{LHL}}$

- RHL: $\lim\limits_{x\to 7^+} \dfrac{x+7}{x-7} = \dfrac{14}{0^+} = +\infty$
- LHL: $\lim\limits_{x\to 7^-} \dfrac{x+7}{x-7} = \dfrac{14}{0^-} = -\infty$

Recall $|x-7| = \begin{cases} x-7 & x-7\geq 0 \\ -(x-7) & x-7<0 \end{cases} = \begin{cases} x-7 & x\geq 7 & \longleftarrow \text{RHL} \\ -(x-7) & x<7 & \longleftarrow \text{LHL} \end{cases}$

7. $\lim\limits_{\mathbf{x}\to\mathbf{2}} \dfrac{\sqrt{\mathbf{x+7}}-\mathbf{3}}{\mathbf{x^2-3x+2}} = \lim\limits_{x\to 2} \dfrac{\sqrt{x+7}-3}{x^2-3x+2} \cdot \left(\dfrac{\sqrt{x+7}+3}{\sqrt{x+7}+3}\right) = \lim\limits_{x\to 2} \dfrac{(x+7)-9}{(x-2)(x-1)(\sqrt{x+7}+3)}$

$= \lim\limits_{x\to 2} \dfrac{x-2}{(x-2)(x-1)(\sqrt{x+7}+3)} = \lim\limits_{x\to 2} \dfrac{1}{(x-1)(\sqrt{x+7}+3)} \overset{\text{L.L.}}{=} \dfrac{1}{1\cdot(\sqrt{9}+3)} = \boxed{\dfrac{1}{6}}$

8. $\lim\limits_{\mathbf{x \to 7}} \dfrac{\mathbf{x+7}}{\mathbf{|x-7|}} = \boxed{+\infty}$ since RHL=LHL

- RHL: $\lim\limits_{x \to 7^+} \dfrac{x+7}{|x-7|} = \lim\limits_{x \to 7^+} \dfrac{x+7}{x-7} = \dfrac{14}{0^+} = +\infty$
- LHL: $\lim\limits_{x \to 7^-} \dfrac{x+7}{|x-7|} = \lim\limits_{x \to 7^-} \dfrac{x+7}{-(x-7)} = \dfrac{14}{-(0^-)} = +\infty$

9. $\lim\limits_{\mathbf{x \to -6}} \dfrac{\mathbf{x+7}}{\mathbf{|x-7|}} = \lim\limits_{x \to -6} \dfrac{x+7}{-(x-7)} \overset{\text{DSP}}{=} \dfrac{-6+7}{-(-6-7)} = \boxed{\dfrac{1}{13}}$

10. $\lim\limits_{\mathbf{x \to -7}} \dfrac{\dfrac{\mathbf{1}}{\mathbf{1-x}} - \dfrac{\mathbf{1}}{\mathbf{8}}}{\mathbf{x+7}} = \lim\limits_{x \to -7} \dfrac{\dfrac{8-(1-x)}{(1-x)\cdot 8}}{x+7} = \lim\limits_{x \to -7} \dfrac{7+x}{(1-x)\cdot 8 \cdot (x+7)} = \lim\limits_{x \to -7} \dfrac{1}{(1-x)\cdot 8}$

$$\overset{\text{DSP}}{=} \frac{1}{(1-(-7))\cdot 8} = \frac{1}{8 \cdot 8} = \boxed{\frac{1}{64}}$$

11. $\lim\limits_{\mathbf{x \to 10}} \dfrac{\mathbf{x+7}}{\mathbf{|x-7|}} = \lim\limits_{x \to 10} \dfrac{x+7}{x-7} \overset{\text{DSP}}{=} \dfrac{10+7}{10-7} = \boxed{\dfrac{17}{3}}$

12. $\lim\limits_{\mathbf{x \to 7}} \dfrac{\mathbf{x^2 - 16x + 63}}{\mathbf{x^2 - 14x + 49}} = \lim\limits_{x \to 7} \dfrac{(x-9)(x-7)}{(x-7)(x-7)} = \lim\limits_{x \to 7} \dfrac{x-9}{x-7}$ $\boxed{\text{DNE since RHL} \neq \text{LHL}}$

- RHL: $\lim\limits_{x \to 7^+} \dfrac{x-9}{x-7} = \dfrac{-2}{0^+} = -\infty$
- LHL: $\lim\limits_{x \to 7^-} \dfrac{x-9}{x-7} = \dfrac{-2}{0^-} = +\infty$

13. $\lim\limits_{\mathbf{x \to 7}} \dfrac{\mathbf{3-x}}{\mathbf{7-x}}$ $\boxed{\text{DNE since RHL} \neq \text{LHL}}$

- RHL: $\lim\limits_{x \to 7^+} \dfrac{3-x}{7-x} = \dfrac{-4}{0^-} = +\infty$
- LHL: $\lim\limits_{x \to 7^-} \dfrac{3-x}{7-x} = \dfrac{-4}{0^+} = -\infty$

14. $\lim\limits_{\mathbf{x \to 7}} \dfrac{\mathbf{3-x}}{\mathbf{(x-7)^2}} = \boxed{-\infty}$ since RHL=LHL

- RHL: $\lim\limits_{x \to 7^+} \dfrac{3-x}{(x-7)^2} = \dfrac{-4}{0^+} = -\infty$
- LHL: $\lim\limits_{x \to 7^-} \dfrac{3-x}{(x-7)^2} = \dfrac{-4}{0^+} = -\infty$

15. $\lim\limits_{\mathbf{x}\to\mathbf{7}} \dfrac{\mathbf{x^2-16x+63}}{\mathbf{x^2-49}} = \lim\limits_{x\to 7} \dfrac{(x-7)(x-9)}{(x-7)(x+7)} = \lim\limits_{x\to 7} \dfrac{x-9}{x+7} \overset{\text{DSP}}{=} \dfrac{-2}{14} = \boxed{-\dfrac{1}{7}}$

16. $\lim\limits_{\mathbf{x}\to\mathbf{7}} \dfrac{\mathbf{7-x}}{\sqrt{\mathbf{x+42}}-\mathbf{7}} = \lim\limits_{x\to 7} \dfrac{7-x}{\sqrt{x+42}-7} \cdot \left(\dfrac{\sqrt{x+42}+7}{\sqrt{x+42}+7}\right) = \lim\limits_{x\to 7} \dfrac{(7-x)(\sqrt{x+42}+7)}{(\sqrt{x+42}-7)\cdot(\sqrt{x+42}+7)}$

$= \lim\limits_{x\to 7} \dfrac{(7-x)(\sqrt{x+42}+7)}{x+42-49} = \lim\limits_{x\to 7} \dfrac{(7-x)(\sqrt{x+42}+7)}{x-7} = \lim\limits_{x\to 7} -(\sqrt{x+42}+7) \overset{\text{L.L.}}{=} \boxed{-14}$

17. $\lim\limits_{\mathbf{x}\to\mathbf{7}} \dfrac{\mathbf{x-7}}{|\mathbf{x-7}|}$ $\boxed{\text{DNE since RHL} \neq \text{LHL}}$

- RHL: $\lim\limits_{x\to 7^+} \dfrac{x-7}{|x-7|} = \lim\limits_{x\to 7^+} \dfrac{x-7}{x-7} = 1$
- LHL: $\lim\limits_{x\to 7^-} \dfrac{x-7}{|x-7|} = \lim\limits_{x\to 7^-} \dfrac{x-7}{-(x-7)} = -1$

18. $\lim\limits_{\mathbf{x}\to\mathbf{7}} \dfrac{\mathbf{1}}{|\mathbf{x-7}|} = \boxed{+\infty}$ since RHL=LHL

- RHL: $\lim\limits_{x\to 7^+} \dfrac{1}{|x-7|} = \lim\limits_{x\to 7^+} \dfrac{1}{x-7} = \dfrac{1}{0^+} = +\infty$
- LHL: $\lim\limits_{x\to 7^-} \dfrac{1}{|x-7|} = \lim\limits_{x\to 7^-} \dfrac{1}{-(x-7)} = \dfrac{1}{-0^-} = +\infty$