

HOMWORK #23

Review Packet for Exam #3

Due Wednesday December 4 at the beginning of class.

Critical Numbers

1. Find critical numbers for the function $f(x) = x\sqrt{6-x}$.
2. Find critical numbers for the function $f(x) = x^{\frac{5}{4}} - 5x^{\frac{1}{4}}$.

Absolute Extreme Values

3. Find the absolute maximum and absolute minimum values of

$$h(x) = \frac{x^2 - 1}{x^2 + 1} \text{ on } [-1, 3].$$

4. Find the absolute maximum and absolute minimum values of

$$f(x) = (x - 4)^2(x + 2)^2 \text{ on } [0, 5].$$

Related Rates

5. A conical reservoir, 12 ft. deep and also 12 ft. across the top is being filled with water at the rate of 5 cubic feet per minute. How fast is the water rising when it is 4 feet deep?
6. A kite 100 feet high is being blown horizontally at 8 feet per second. When there are 300 feet of string out, how fast is the string running out?
7. A waterskier skis up over the ramp at a speed of 30 ft./sec. The 100 ft. ramp slopes straight from no height at one end to 20 feet on the other end. How fast is she rising vertically just as she leaves the ramp?
8. Two trucks leave a depot at the same time. Truck A travels east at 40 miles per hour, while Truck B travels north at 30 miles per hour. How fast is the distance between the trucks changing 60 minutes after leaving the depot?

Limits Evaluate the following limits. Please show your work.

9. $\lim_{x \rightarrow -\infty} \frac{x^3 - 2x}{4x^3 + 1}$
10. $\lim_{x \rightarrow \infty} \frac{x^3 + 1}{x^7 + 2x^{\frac{7}{2}}}$
11. $\lim_{x \rightarrow \infty} \frac{x^6 + 1}{x^3 + 9x^2 + 7}$

Curve Sketching For each of the following functions, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

12. $f(x) = \frac{1}{x^2 - 9}$
13. $f(x) = \frac{x^3 + 6x^2 - 40}{(x + 2)^3}$.

Take my word for it that (you do NOT have to compute these)

$$f'(x) = \frac{24(x + 5)}{(x + 2)^4} \text{ and } f''(x) = \frac{-72(x + 6)}{(x + 2)^5}.$$

Position, Velocity, Acceleration

14. Suppose that Dan throws a ball, from the ground, straight upward in the air with an initial velocity of 128 meters per second. The ball reaches a height of $\mathbf{s(t) = 128t - 16t^2}$ feet in t seconds. Suppose Sam is lying on the ground under the ball. Answer the following questions:
 - (a) What is the maximum height the ball reaches?
 - (b) What is the ball's velocity at time $t = 5$?
 - (c) What is the ball's acceleration at time $t = 5$?
 - (d) At what time will the ball hit Sam on the ground?
 - (e) What is the ball's velocity when it hits Sam?
 - (f) What is the ball's acceleration when it hits Sam?
15. RETALIATION! When Dan saw that the ball actually hit Sam, he ran away, up a tree. Dan climbed up the tree exactly 155 feet (above the ground). Revenge was necessary! Sam managed to throw the ball upward at Dan with an initial velocity of 96 feet per second. This time the ball reaches a height of $\mathbf{s(t) = 96t - 16t^2}$ feet in t seconds.

Does the ball hit Dan? If it doesn't, explain why. If it does, explain why. Show your work.