

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.
- Please *show* all of your work and *justify* all of your answers.

Critical Numbers

1. Find critical numbers for the function $f(x) = \frac{x^2 + 1}{x - 3}$.
2. Find critical numbers for the function $f(x) = 3x^{\frac{2}{3}} - \frac{x}{4}$.

Absolute Extreme Values

3. Find the absolute maximum and absolute minimum values of

$$F(x) = \frac{1}{x^2 - 16} \quad \text{on} \quad [-1, 3].$$

4. Find the absolute maximum and absolute minimum values of

$$G(x) = (x - 3)^2(x + 2)^3 \quad \text{on} \quad [0, 4].$$

Related Rates

5. Suppose that garbage is being compacted in the shape of a cube, so that the volume of the cube is shrinking at 6 cubic feet per second. How fast is the length of the edge of the cube changing when the length of the edge has been compacted to 2 feet?
6. Suppose a train depot is 10 feet directly south from the train track. The train is travelling east at 15 feet per second. How fast is the distance between the train and train depot station increasing when 2 seconds has passed since the train passed directly in front of the depot?

Limits at Infinity

7. Compute each of the following limits at infinity. Please show your work.

(a) $\lim_{x \rightarrow \infty} \frac{15 - 3x^2}{7x^2 + 20}$

(b) $\lim_{x \rightarrow \infty} \frac{x^9 + 8x^7 + 6x^5 + 4}{3x^2 + 1}$

(c) $\lim_{x \rightarrow \infty} \frac{6x^7 - 7x^6 + 2}{7x^8 + 8x^7 + 2}$

Curve Sketching For each of the following functions, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

8. $f(x) = \frac{x^2 - 16}{x^2 - 9}$

9. $f(x) = \frac{-x^2 + x + 2}{x^2 - 2x + 1}$. Take my word for it that (you do NOT have to compute these)

$$f'(x) = \frac{x - 5}{(x - 1)^3} \text{ and } f''(x) = \frac{-2x + 14}{(x - 1)^4}.$$

Position, Velocity, Acceleration

10. A ball is thrown straight upward from the ground with initial velocity $v_0 = 128$ feet per second. The height of the ball at time t is given by the position function $s(t) = -16t^2 + 128t$ feet in t seconds.

Answer the following questions:

- (a) What is the maximum height attained by the ball?
(b) Find the velocity with which the ball hits the ground upon its return, at impact.