Practice Exam #2 Math 105 Fall 2013

1. (a) $\lim_{x\to 7} g(x) = g(7) = \boxed{-3}$ The first equality is true because $g(x)$ was assumed to be continuous at $x=7$. The second equality was given.

(b). $g \circ f(5) = g(f(5)) = g(7) = \boxed{-3}$ Each value was given in assumptions.

(c) $\boxed{\text{No}}$ $f(7) \neq 5$ because by assumption, $f(x)$ was assumed to NOT be continuous at $x=7$, so by definition of continuity

$$\underbrace{5}_{\text{given}} = \lim_{x\to 7} f(x) \neq f(7).$$

2.(a) $y = \frac{5}{6}x + x^{5/6} + x^{-5/6} + (5x+6)^{1/2} + (5x+6)^{-1/2}$

$$y' = \boxed{\frac{5}{6} + \frac{5}{6}x^{-1/6} - \frac{5}{6}x^{-11/6} + \frac{1}{2}(5x+6)^{-1/2}(5) - \frac{1}{2}(5x+6)^{-3/2}(5)}$$

(b) $y = \left(\dfrac{2\sqrt{x} + x^3}{x^{2/3} + \frac{2}{3}x}\right)^{2/3}$

$$y' = \boxed{\frac{2}{3}\left(\frac{2\sqrt{x}+x^3}{x^{2/3}+\frac{2}{3}x}\right)^{-1/3} \cdot \left[\frac{\left(x^{2/3}+\frac{2}{3}x\right)\left[2\cdot\frac{1}{2\sqrt{x}} + 3x^2\right] - \left(2\sqrt{x}+x^3\right)\left[\frac{2}{3}x^{-1/3} + \frac{2}{3}\right]}{\left(x^{2/3}+\frac{2}{3}x\right)^2}\right]}$$

$$3x^{-2} \qquad 2x^{-3}$$

2(c) $$f(x) = \left(\frac{3}{x^2} - \frac{2}{x^3}\right)^9 \sqrt{5-x^2}$$

$$\boxed{f'(x) = \left(\frac{3}{x^2} - \frac{2}{x^3}\right)^9 \left(\frac{1}{2\sqrt{5-x^2}}\right)(-2x) + \sqrt{5-x^2}\,(9)\left(\frac{3}{x^3} - \frac{2}{x^3}\right)^8 \left[-6x^{-3} + 6x^{-4}\right]}$$

(d) $$y = \frac{\frac{1}{x} - 6x^3}{\sqrt{7x+x^8}}$$

$$\boxed{y' = \frac{\sqrt{7x+x^8}\left(-\frac{1}{x^2} - 18x^2\right) - \left(\frac{1}{x} - 6x^3\right)\frac{1}{2\sqrt{7x+x^8}}(7+8x^7)}{7x+x^8}}$$

3. $$y = \left(6x + \sqrt{8+x^2}\right)^{3/2} \qquad y(1) = \left(6+\sqrt{9}\right)^{3/2} = \left(\sqrt{9}\right)^3 = 3^3 = 27$$

$$y' = \frac{3}{2}\left(6x + \sqrt{8+x^2}\right)^{1/2} \cdot \left[6 + \frac{1}{\cancel{2}\sqrt{8+x^2}}(\cancel{2}x)\right]$$

$$y'(1) = \frac{3}{2}\sqrt{6 + \sqrt{9}_{\to 3}} \cdot \left[6 + \frac{1}{\sqrt{9}_{\to 3}}\right]$$

$$= \frac{3}{2}\sqrt{9}^{\,\to 3} \cdot \left[\frac{18}{3} + \frac{1}{3}\right]$$

$$= \frac{\cancel{9}^{\,3}}{2} \cdot \frac{19}{\cancel{3}} = \frac{57}{2} \leftarrow \text{specific slope}$$

Point $(1, 27)$ Slope $y'(1) = 57/2$

Equation of Tangent Line

$$y - 27 = 57/2\,(x-1) \qquad y - 27 = \frac{57}{2}x - \frac{57}{2} \qquad \boxed{y = \frac{57}{2}x - \frac{3}{2}}$$

$$+27 \qquad +27 \to \frac{54}{2}$$

4. $f(x) = \dfrac{3x-1}{2-5x}$

(1) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{3(x+h)-1}{2-5(x+h)} - \dfrac{3x-1}{2-5x}}{h}$

$$= \lim_{h\to 0} \frac{(2-5x)(3x+3h-1) - (3x-1)(2-5x-5h)}{(2-5x-5h)(2-5x)} \cdot \left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{6x+6h-2-15x^2-15xh+5x-(6x-15x^2-15xh-2+5x+5h)}{(2-5x-5h)(2-5x)} \cdot \left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{6x}+\boxed{6h}\cancel{-2}\cancel{-15x^2}\cancel{-15xh}\cancel{+5x}\cancel{-6x}\cancel{+15x^2}\cancel{+15xh}\cancel{+2}\cancel{-5x}\boxed{-5h}}{(2-5x-5h)(2-5x)} \cdot \left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{(2-5x-\underbrace{5h}_{0})(2-5x)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{1}{(2-5x)^2}}$$

(2) $f'(x) = \dfrac{(2-5x)(3)-(3x-1)(-5)}{(2-5x)^2} = \dfrac{6-\cancel{15x}+\cancel{15x}-5}{(2-5x)^2} = \boxed{\dfrac{1}{(2-5x)^2}}$

Simplify $f'(x) = (2-5x)^{-2}$

$\Rightarrow \quad f''(x) = -2(2-5x)^{-3}(-5) = \boxed{\dfrac{10}{(2-5x)^3}}$

5. $f(x) = (5+3x^2)^8 (7-x^2)^3$

$$f'(x) = (5+3x^2)^8 \cdot 3(7-x^2)^2(-2x) + (7-x^2)^3 \cdot 8(5+3x^2)^7(6x)$$

$$= x(5+3x^2)^7(7-x^2)^2 6\left[\underbrace{(-1)(5+3x^2) + 8(7-x^2)}_{-5-3x^2+56-8x^2}\right]$$

$$= x(5+3x^2)^7(7-x^2)^2 \cdot 6\left[-11x^2+51\right] \underset{=}{\text{set}} 0$$

$\Rightarrow \boxed{x=0}$ or $5+3x^2=0$ or $7-x^2=0$ or $-11x^2+51=0$

$3x^2=-5$

No Solution

$x^2=7$

$\boxed{x=\pm\sqrt{7}}$

$x^2 = 51/11$

$\boxed{x=\pm\sqrt{51/11}}$

6. $f(x) = \dfrac{5x}{1+x}$

$$f'(x) = \frac{(1+x)(5) - 5x(1)}{(1+x)^2} = \frac{1}{(1+x)^2}$$

$$f'(0) = \frac{1}{(1+0)^2} = \boxed{1}$$

$$f'(1) = \frac{1}{(1+1)^2} = \boxed{\frac{1}{4}}$$

$$f'(2) = \frac{1}{(1+2)^2} = \boxed{\frac{1}{9}}$$

7. (a) $$\frac{x}{y+1} = x^2 - y^2$$

$$\frac{d}{dx}\left[\frac{x}{y+1}\right] = \frac{d}{dx}\left[x^2 - y^2\right]$$ Implicitly differentiate both sides

$$\frac{(y+1)(1) - x \cdot \frac{dy}{dx}}{(y+1)^2} = 2x - 2y\frac{dy}{d}$$ plug in $x=1,\ y=0$

$$\frac{1 - \frac{dy}{dx}}{1^2} = 2 - 0$$

$$1$$

$$1 - \frac{dy}{dx} = 2$$

$$-\frac{dy}{dx} = 1 \implies \left.\frac{dy}{dx}\right|_{(1,0)} = -1$$ specific slope.

Equation of Tangent Line.

$$y - 0 = -1(x-1)$$

$$\boxed{y = -x + 1}$$

7(b) $4(x+y)^2 = x^2y^2$

$$\frac{d}{dx}\left[4(x+y)^2\right] = \frac{d}{dx}\left[x^2y^2\right]$$ implicitly differentiate.

$$8(x+y)\left[1+\frac{dy}{dx}\right] = x^2 \cdot 2y\,\frac{dy}{dx} + y^2(2x)$$ plug in $(-2,1)$

$$8(-2+1)\left[1+\frac{dy}{dx}\right] = (4)\cdot 2\,\frac{dy}{dx} - 4$$

$$-8\left[1+\frac{dy}{dx}\right] = 8\,\frac{dy}{dx} - 4$$

$$-8 - 8\frac{dy}{dx} = 8\,\frac{dy}{dx} - 4$$

$$-4 = 16\frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{(-2,1)} = \frac{-1}{4}$$ ← specific slope

Equation of Tangent Line.

$$y-1 = -\tfrac{1}{4}(x-(-2))$$

$$y-1 = -\tfrac{1}{4}(x+2)$$

$$y-1 = -\tfrac{1}{4}x - \tfrac{1}{2}$$

$$\quad +1 \qquad\qquad +1$$

$$\boxed{y = -\tfrac{1}{4}x + \frac{1}{2}}$$