This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

Please show all of your work and justify all of your answers.

1. Suppose that $f$ and $g$ are functions, and

- $\lim_{x \to 7} f(x) = 5$
- $f(5) = 7$
- $g(7) = -3$
- $g(x)$ is continuous at $x = 7$
- $f(x)$ is NOT continuous at $x = 7$

Evaluate the following quantities and fully justify your answers. Do not just put down a value:

(a) $\lim_{x \to 7} g(x) =$ 

(b) $g \circ f(5) =$ 

(c) Does $f(7) = 5$? Justify.

2. Compute the derivative of each of the following functions. For these problems, you do NOT need to simplify your derivative.

(a) $y = \frac{5}{6} x + x^\frac{5}{6} + \frac{1}{x^\frac{5}{6}} + \sqrt{5x + 6} + \frac{1}{\sqrt{5x + 6}}$

(b) $y = \left( \frac{2\sqrt{x} + x^3}{x^\frac{2}{3} + \frac{2}{3}x} \right) ^ \frac{2}{3}$

(c) $f(x) = \left( \frac{3}{x^2} - \frac{2}{x^3} \right) ^ 9 \sqrt{5 - x^2}$

(d) $y = \frac{1}{x} - 6x^3$

$\sqrt{7x + x^8}$
3. Find the equation of the tangent line to this curve \( y = \left(6x + \sqrt{8 + x^2}\right)^3 \) at the point where \( x = 1 \).

4. Compute the derivative of \( f(x) = \frac{3x - 1}{2 - 5x} \) two different ways:
   
   - First use the limit definition of the derivative.
   - Second use the Chain Rule.

   Next simplify your answer from the first part. Then compute the second derivative \( f''(x) \). Simplify your final answer to a single fraction.

5. Find all \( x \)-coordinates at which the graph of the following function has horizontal tangent lines. Please simplify your derivative first. Why?

\[
f(x) = (5 + 3x^2)^8(7 - x^2)^3
\]

6. Consider \( f(x) = \frac{5x}{1 + x} \). Compute \( f'(0), f'(1), \) and \( f'(2) \).

7. Implicit Differentiation Practice

   (a) Consider the equation \( \frac{x}{y + 1} = x^2 - y^2 \). Compute the equation of the tangent line at the point \((1, 0)\).

   (b) Find the equation of the tangent line to the curve \( 4(x + y)^2 = x^2 y^2 \) at the point \((-2, 1)\).