Math 105 **Practice Exam** #2 **Fall 2013** Compact Version

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers.

1. Suppose that f and g are functions, **and**

   • $\lim_{x\to 7} f(x) = 5$ • $f(5) = 7$ • $g(7) = -3$

   • $g(x)$ is continuous at $x = 7$. • $f(x)$ is NOT continuous at $x = 7$

   Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

   (a) $\lim_{x\to 7} g(x) =$

   (b) $g \circ f(5) =$

   (c) Does $f(7) = 5$? Justify.

2. Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

   (a) $y = \dfrac{5}{6}x + x^{\frac{5}{6}} + \dfrac{1}{x^{\frac{5}{6}}} + \sqrt{5x+6} + \dfrac{1}{\sqrt{5x+6}}$

   (b) $y = \left(\dfrac{2\sqrt{x} + x^3}{x^{\frac{2}{3}} + \dfrac{2}{3}x}\right)^{\frac{2}{3}}$

   (c) $f(x) = \left(\dfrac{3}{x^2} - \dfrac{2}{x^3}\right)^9 \sqrt{5 - x^2}$

   (d) $y = \dfrac{\dfrac{1}{x} - 6x^3}{\sqrt{7x + x^8}}$

3. Find the equation of the tangent line to this curve $y = \left(6x + \sqrt{8 + x^2}\right)^{\frac{3}{2}}$ at the point where $x = 1$.

4. Compute the derivative of $f(x) = \dfrac{3x - 1}{2 - 5x}$ **two** different ways:

   - First use the **limit definition of the derivative**.
   - Second use the Chain Rule.

   Next simplify your answer from the first part. Then compute the second derivative $f''(x)$. Simplify your final answer to a single fraction.

5. Find **all** x-coordinates at which the graph of the following function has horizontal tangent lines. Please **simplify** your derivative first. Why?

$$f(x) = (5 + 3x^2)^8(7 - x^2)^3$$

6. Consider $f(x) = \dfrac{5x}{1 + x}$. Compute $f'(0)$, $f'(1)$, and $f'(2)$,

7. Implicit Differentiation Practice

   (a) Consider the equation $\dfrac{x}{y + 1} = x^2 - y^2$. Compute the equation of the tangent line at the point $(1, 0)$.

   (b) Find the equation of the tangent line to the curve $4(x + y)^2 = x^2y^2$ at the point $(-2, 1)$.