

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Simplify your answers if required.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points] Suppose that  $f$  and  $g$  are functions, **and**

- $\lim_{x \rightarrow 3} f(x) = 9$
- $g(7) = -6$
- $\lim_{x \rightarrow 4} f(x) = 7$
- $g(x)$  is continuous at  $x = 7$
- $f(3) = -5$
- $f(x)$  is continuous at  $x = 4$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a)  $f(4) =$

(b)  $\lim_{x \rightarrow 7} g(x) =$

(c) Compute  $g \circ f(4) =$

(d) Is  $f(x)$  continuous at  $x = 3$ ? Why or why not? Use math notation.

2. [35 Points] Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(a)  $y = \frac{5}{6}x + x^{\frac{5}{6}} + \sqrt{5x+6} + \frac{1}{\sqrt{5x+6}}$ .

(b)  $y = \left(\frac{x}{3} + \frac{5}{x^8}\right)^9$

(c)  $f(x) = \left(x^2 - \frac{5}{x^2}\right)(3x + \sqrt{x})$

(d)  $y = \frac{1}{\sqrt{x^2 - 5x + 3}}$

(e)  $y = \left(\frac{1}{x^3} + 7x\right)^{\frac{5}{7}} \left(x^4 - \frac{1}{x^7}\right)^{-5}$

(f)  $f(x) = \left(\frac{2\sqrt{x} + x^3}{x^{\frac{2}{3}} + \frac{2}{3}x}\right)^{\frac{2}{3}}$

3. [10 Points] Find the equation of the tangent line to this curve  $y = \sqrt{x + (x^2 + 1)^3}$  at the point where  $x = 1$ .

4. [15 Points] Consider the function  $f(x) = \frac{7x + 3}{1 - 5x}$ .

(a) Compute the derivative of  $f$  using the **limit definition of the derivative**.

(b) Compute the derivative of  $f$  using the Quotient Rule.

(c) Compute the second derivative  $f''(x)$ .

5. [10 Points] Find **all**  $x$ -coordinates at which the graph of the function

$$f(x) = (4x + 1)^4(7 - 3x)^8$$

has horizontal tangent lines.

6. [10 Points] Find the equation of the line tangent to the curve  $x^3 + x^2y = 6 - 4y^2$  at the point  $(1, 1)$ .

7. [10 Points] For each of the following functions, compute the derivative of  $f$  **and simplify** your derivative as much as possible **into a single fraction**.

(a)  $f(x) = \frac{x}{\sqrt{x^2 + 1}}$ .

(b)  $f(x) = \left(\frac{x^2 + 1}{x^2 - 1}\right)^3$

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## OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute  $\lim_{x \rightarrow 0} \frac{|x - 1| - |x + 1| - |x|}{|x| + |2 - x| - |x + 2|}$