

Name: _____

Amherst College
DEPARTMENT OF MATHEMATICS
Math 105
Midterm Exam #2
November 1, 2013

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

- Simplify your answers if required.

- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		10
2		35
3		10
4		15
5		10
6		10
7		10
Total		100

1. [10 Points] Suppose that f and g are functions, **and**

- $\lim_{x \rightarrow 3} f(x) = 9$
- $g(7) = -6$
- $\lim_{x \rightarrow 4} f(x) = 7$
- $g(x)$ is continuous at $x = 7$
- $f(3) = -5$
- $f(x)$ is continuous at $x = 4$

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a) $f(4) =$

(b) $\lim_{x \rightarrow 7} g(x) =$

(c) Compute $g \circ f(4) =$

(d) Is $f(x)$ continuous at $x = 3$? Why or why not? Use math notation.

2. [35 Points] Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(a) $y = \frac{5}{6}x + x^{\frac{5}{6}} + \sqrt{5x+6} + \frac{1}{\sqrt{5x+6}}$.

(b) $y = \left(\frac{x}{3} + \frac{5}{x^8}\right)^9$

(c) $f(x) = \left(x^2 - \frac{5}{x^2}\right)(3x + \sqrt{x})$

2. (Continued) Compute the derivative of each of the following functions. For these problems, you do **NOT** need to simplify your derivative.

(d) $y = \frac{1}{\sqrt{x^2 - 5x + 3}}$

(e) $y = \left(\frac{1}{x^3} + 7x\right)^{\frac{5}{7}} \left(x^4 - \frac{1}{x^7}\right)^{-5}$

(f) $f(x) = \left(\frac{2\sqrt{x} + x^3}{x^{\frac{2}{3}} + \frac{2}{3}x}\right)^{\frac{2}{3}}$

3. [10 Points] Find the equation of the tangent line to this curve $y = \sqrt{x + (x^2 + 1)^3}$ at the point where $x = 1$.

4. [15 Points] Consider the function $f(x) = \frac{7x + 3}{1 - 5x}$.

(a) Compute the derivative of f using the **limit definition of the derivative**.

(b) Compute the derivative of f using the Quotient Rule.

(c) Compute the second derivative $f''(x)$.

5. [10 Points] Find **all** x -coordinates at which the graph of the function

$$f(x) = (4x + 1)^4(7 - 3x)^8$$

has horizontal tangent lines.

6. [10 Points] Find the equation of the line tangent to the curve $x^3 + x^2y = 6 - 4y^2$ at the point $(1, 1)$.

7. [10 Points] For each of the following functions, compute the derivative of f **and simplify** your derivative as much as possible **into a single fraction**.

(a) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$.

(b) $f(x) = \left(\frac{x^2 + 1}{x^2 - 1}\right)^3$

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\lim_{x \rightarrow 0} \frac{|x-1| - |x+1| - |x|}{|x| + |2-x| - |x+2|}$