## HOMEWORK #9

Review Packet for Exam #1

**Due Wednesday October 2** at the beginning of class.

### Limit Practice Problems

Evaluate the following limits. Be clear if the limit equals a finite value, Does Not Exist, or is $+\infty$ or $-\infty$. Always justify your work:

1. $\lim_{w\to 0} \frac{16}{w} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

   RHL: $\lim_{w\to 0^+} \frac{16}{w} = \frac{16}{0^+} = +\infty$

   LHL: $\lim_{w\to 0^-} \frac{16}{w} = \frac{16}{0^-} = -\infty$

2. $\lim_{t\to 2} \frac{3-t}{t-2} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

   RHL: $\lim_{t\to 2^+} \frac{3-t}{t-2} = \frac{3-2}{0^+} = \frac{1}{0^+} = +\infty$

   LHL: $\lim_{t\to 2^-} \frac{3-t}{t-2} = \frac{3-2}{0^-} = \frac{1}{0^-} = -\infty$

3. $\lim_{t\to 2} \frac{3-t}{(t-2)^2} = \boxed{+\infty}$ since RHL = LHL

   RHL: $\lim_{t\to 2^+} \frac{3-t}{(t-2)^2} = \frac{3-2}{(0^+)^2} = \frac{1}{0^+} = +\infty$

   LHL: $\lim_{t\to 2^-} \frac{3-t}{(t-2)^2} = \frac{3-2}{(0^-)^2} = \frac{1}{0^+} = +\infty$

4. $\lim_{x\to 4} \frac{(x+2)^2}{x^2-3x-4} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

   RHL: $\lim_{x\to 4^+} \frac{(x+2)^2}{x^2-3x-4} = \lim_{x\to 4^+} \frac{(x+2)^2}{(x-4)(x+1)} = \frac{(4+2)^2}{0^+(4+1)} = \frac{36}{0^+(5)} = +\infty$

   LHL: $\lim_{x\to 4^-} \frac{(x+2)^2}{x^2-3x-4} = \lim_{x\to 4^-} \frac{(x+2)^2}{(x-4)(x+1)} = \frac{(4+2)^2}{0^-(4+1)} = \frac{36}{0^-(5)} = -\infty$

5. $\lim_{x\to 4} \frac{x-4}{x^2-3x-4} \overset{\frac{0}{0}}{=} \lim_{x\to 4} \frac{x-4}{(x-4)(x+1)} = \lim_{x\to 4} \frac{1}{x+1} \overset{\text{DSP}}{=} \boxed{\frac{1}{5}}$

6. $\lim_{x\to 4} \frac{x^2-2x-8}{x^2-3x-4} \overset{\frac{0}{0}}{=} \lim_{x\to 4} \frac{(x-4)(x+2)}{(x-4)(x+1)} = \lim_{x\to 4} \frac{x+2}{x+1} \overset{\text{DSP}}{=} \frac{4+2}{4+1} = \boxed{\frac{6}{5}}$

7. $\lim_{x\to 1} \frac{x^2-4x-12}{x^2-3x-18} = \frac{1-4-12}{1-3-18} \overset{\text{DSP}}{=} \frac{-15}{-20} = \boxed{\frac{3}{4}}$

8. $\lim_{x\to 0} \frac{x^2-4x-12}{x^2-3x-18} \overset{\text{DSP}}{=} \frac{-12}{-18} = \boxed{\frac{2}{3}}$

9. $\lim_{x\to -3} \dfrac{x+2}{x+3} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to -3^+} \dfrac{x+2}{x+3} = \dfrac{-1}{0^+} = -\infty$

LHL: $\lim_{x\to -3^-} \dfrac{x+2}{x+3} = \dfrac{-1}{0^-} = +\infty$

10. $\lim_{x\to -2} \dfrac{x^2-4x-12}{x^2-3x-18} \overset{\text{DSP}}{=} \dfrac{4+8-12}{4+6-18} = \dfrac{0}{-8} = \boxed{0}$

11. $\lim_{x\to 0} \dfrac{x^2-4x}{x^2-7x}^{\frac{0}{0}} = \lim_{x\to 0} \dfrac{x(x-4)}{x(x-7)} = \lim_{x\to 0} \dfrac{x-4}{x-7} \overset{\text{DSP}}{=} \dfrac{-4}{-7} = \dfrac{4}{7}$

12. $\lim_{x\to 3} \dfrac{x^2-9}{|x-3|} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to 3^+} \dfrac{x^2-9}{|x-3|} = \lim_{x\to 3^+} \dfrac{(x-3)(x+3)}{x-3} = \lim_{x\to 3^+} x+3 \overset{\text{DSP}}{=} 6$

LHL: $\lim_{x\to 3^-} \dfrac{x^2-9}{|x-3|} = \lim_{x\to 3^-} \dfrac{(x-3)(x+3)}{-(x-3)} = \lim_{x\to 3^-} -(x+3) \overset{\text{DSP}}{=} -6$

Recall: $|x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$

13. $\lim_{x\to -5} \dfrac{x^2+6x+5}{|x+5|} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to (-5)^+} \dfrac{x^2+6x+5}{|x+5|} = \lim_{x\to (-5)^+} \dfrac{(x+5)(x+1)}{x+5} = \lim_{x\to (-5)^+} x+1 \overset{\text{DSP}}{=} -4$

LHL: $\lim_{x\to (-5)^-} \dfrac{x^2+6x+5}{|x+5|} = \lim_{x\to (-5)^-} \dfrac{(x+5)(x+1)}{-(x+5)} = \lim_{x\to (-5)^-} -(x+1) \overset{\text{DSP}}{=} 4$

Recall: $|x+5| = \begin{cases} x+5 & \text{if } x+5 \geq 0 \\ -(x+5) & \text{if } x+5 < 0 \end{cases} = \begin{cases} x+5 & \text{if } x \geq -5 \\ -(x+5) & \text{if } x < -5 \end{cases}$

14. $\lim_{t\to 1} \dfrac{t^2-1}{t^2-11t+10}^{\frac{0}{0}} = \lim_{t\to 1} \dfrac{(t-1)(t+1)}{(t-10)(t-1)} = \lim_{t\to 1} \dfrac{t+1}{t-10} \overset{\text{DSP}}{=} \dfrac{1+1}{1-10} = \boxed{-\dfrac{2}{9}}$

15. $\lim_{t\to 1} \dfrac{t^2}{t^2+t-1} \overset{\text{DSP}}{=} \dfrac{1}{1+1-1} = \boxed{1}$

16. $\lim_{t\to -1} \dfrac{2009(t^2+6t+5)}{t^2+t}^{\frac{0}{0}} = \lim_{t\to -1} \dfrac{2009(t+5)(t+1)}{t(t+1)} = \lim_{t\to -1} \dfrac{2009(t+5)}{t} \overset{\text{DSP}}{=} \dfrac{2009(4)}{-1} = \boxed{-8036}$

17. $\lim_{x\to 9} \dfrac{x^2-10x+9}{x^2+x-90}^{\frac{0}{0}} = \lim_{x\to 9} \dfrac{(x-9)(x-1)}{(x+10)(x-9)} = \lim_{x\to 9} \dfrac{x-1}{x+10} \overset{\text{DSP}}{=} \dfrac{9-1}{9+10} = \boxed{\dfrac{8}{19}}$

18. $\lim_{t\to 1} t^{500} + t^{400} + t^{300} + t^{200} + t^{100} \overset{\text{DSP}}{=} \boxed{5}$

19. $\displaystyle\lim_{x\to 3}\frac{x^2-x-6}{x^2-2x-3}\overset{\frac{0}{0}}{} = \lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)(x+1)} = \lim_{x\to 3}\frac{x+2}{x+1} \overset{\text{DSP}}{=} \frac{3+2}{3+1} = \boxed{\frac{5}{4}}$

20. $\displaystyle\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}\overset{\frac{0}{0}}{} = \lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}\cdot\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4}$

$\displaystyle= \lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{x-1} = \lim_{x\to 1}\sqrt{x+3}+2 \overset{\text{L.L.}}{=} \boxed{4}$

21. $\displaystyle\lim_{x\to 9}\frac{9x-x^2}{3-\sqrt{x}}\overset{\frac{0}{0}}{} = \lim_{x\to 9}\frac{9x-x^2}{3-\sqrt{x}}\cdot\frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9}\frac{x(9-x)(3+\sqrt{x})}{9-x}$

$\displaystyle= \lim_{x\to 9}x(3+\sqrt{x}) \overset{\text{L.L.}}{=} 9(3+3) = \boxed{54}$

22. $\displaystyle\lim_{x\to -1}\frac{5}{1-x} \overset{\text{DSP}}{=} \boxed{\frac{5}{2}}$

23. $\displaystyle\lim_{x\to 5}\frac{6x}{5-x} = \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\displaystyle\lim_{x\to 5^+}\frac{6x}{5-x} = \frac{30}{0^-} = -\infty$

LHL: $\displaystyle\lim_{x\to 5^-}\frac{6x}{5-x} = \frac{30}{0^+} = +\infty$

24. $\displaystyle\lim_{x\to 2}\frac{x^2-9x+14}{x^2-4x+4}\overset{\frac{0}{0}}{} = \lim_{x\to 2}\frac{(x-2)(x-7)}{(x-2)(x-2)} = \lim_{x\to 2}\frac{x-7}{x-2} \quad \boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\displaystyle\lim_{x\to 2^+}\frac{x-7}{x-2} = \lim_{x\to 2^+}\frac{-5}{0^+} = -\infty$

LHL: $\displaystyle\lim_{x\to 2^-}\frac{x-7}{x-2} = \lim_{x\to 2^-}\frac{x-7}{x-2} = \lim_{x\to 2^-}\frac{-5}{0^-} = +\infty$

25. $\displaystyle\lim_{x\to 2}\frac{x^2-4}{|x-2|} = \boxed{\text{DNE b/c RHL} \neq \text{LHL}}$.

RHL: $\displaystyle\lim_{x\to 2^+}\frac{x^2-4}{|x-2|} = \lim_{x\to 2^+}\frac{x^2-4}{x-2} = \lim_{x\to 2^+}\frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^+}x+2 \overset{\text{DSP}}{=} 4$

LHL: $\displaystyle\lim_{x\to 2^-}\frac{x^2-4}{|x-2|} = \lim_{x\to 2^-}\frac{x^2-4}{-(x-2)} = \lim_{x\to 2^-}\frac{(x-2)(x+2)}{-(x-2)} = \lim_{x\to 2^-}-(x+2) \overset{\text{DSP}}{=} -4$

Recall: $|x-2| = \begin{cases} x-2 & \text{if } x-2\geq 0 \\ -(x-2) & \text{if } x-2<0 \end{cases} = \begin{cases} x-2 & \text{if } x\geq 2 \\ -(x-2) & \text{if } x<2 \end{cases}$

26. $\displaystyle\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x^2-x-6}\overset{\frac{0}{0}}{} = \lim_{x\to 3}\frac{\sqrt{x+6}-3}{x^2-x-6} = \lim_{x\to 3}\frac{x+6-9}{(x-3)(x+2)(\sqrt{x+6}+3)}$

$\displaystyle= \lim_{x\to 3}\frac{x-3}{(x-3)(x+2)(\sqrt{x+6}+3)} = \lim_{x\to 3}\frac{1}{(x+2)(\sqrt{x+6}+3)} \overset{\text{L.L.}}{=} \frac{1}{5(3+3)} = \boxed{\frac{1}{30}}$

27. $\displaystyle\lim_{x\to7}\frac{\frac{1}{7}-\frac{1}{x}}{x-7}\overset{\frac{0}{0}}{} = \lim_{x\to7}\frac{\frac{x}{7x}-\frac{7}{7x}}{x-7} = \lim_{x\to7}\frac{\frac{x-7}{7x}}{x-7} = \lim_{x\to7}\frac{x-7}{7x}\cdot\frac{1}{x-7} = \lim_{x\to7}\frac{x-7}{(7x)(x-7)}$

$\displaystyle = \lim_{x\to7}\frac{1}{7x} \overset{\text{DSP}}{=} \boxed{\frac{1}{49}}$

28. $\displaystyle\lim_{x\to-6}\frac{\frac{1}{2-x}-\frac{1}{8}}{x+6}\overset{\frac{0}{0}}{} = \lim_{x\to-6}\frac{\frac{8-(2-x)}{(2-x)(8)}}{x+6} = \lim_{x\to-6}\frac{\frac{6+x}{(2-x)(8)}}{x+6}$

$\displaystyle = \lim_{x\to-6}\frac{6+x}{(2-x)(8)(x+6)} = \lim_{x\to-6}\frac{1}{(2-x)(8)} \overset{\text{DSP}}{=} \boxed{\frac{1}{64}}$

29. $\displaystyle\lim_{x\to3}\frac{\sqrt{x+1}-2}{3-x}\overset{\frac{0}{0}}{} = \lim_{x\to3}\frac{\sqrt{x+1}-2}{3-x}\cdot\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right) = \lim_{x\to3}\frac{x+1-4}{(3-x)(\sqrt{x+1}+2)}$

$\displaystyle = \lim_{x\to3}\frac{x-3}{(3-x)(\sqrt{x+1}+2)} = \lim_{x\to3}\frac{x-3}{-(x-3)(\sqrt{x+1}+2)}$

$\displaystyle = \lim_{x\to3}\frac{-1}{\sqrt{x+1}+2} \overset{\text{L.L.}}{=} \frac{-1}{2+2} = \boxed{-\frac{1}{4}}$

30. $\displaystyle\lim_{x\to7}\frac{x^2-49}{2-\sqrt{x-3}}\overset{\frac{0}{0}}{} = \lim_{x\to7}\frac{x^2-49}{2-\sqrt{x-3}}\cdot\left(\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}\right) = \lim_{x\to7}\frac{(x^2-49)(2+\sqrt{x-3})}{4-(x-3)}$

$\displaystyle = \lim_{x\to7}\frac{(x-7)(x+7)(2+\sqrt{x-3})}{7-x} = \lim_{x\to7}\frac{(x-7)(x+7)(2+\sqrt{x-3})}{-(x-7)}$

$\displaystyle = \lim_{x\to7}-(x+7)(2+\sqrt{x-3}) \overset{\text{L.L.}}{=} -(14)(2+2) = \boxed{-56}$

31. $\displaystyle\lim_{x\to5}\frac{\frac{1}{\sqrt{x+20}}-\frac{1}{5}}{x-5}\overset{\frac{0}{0}}{} = \lim_{x\to5}\frac{\frac{5-\sqrt{x+20}}{5(\sqrt{x+20})}}{x-5} = \lim_{x\to5}\frac{5-\sqrt{x+20}}{5(\sqrt{x+20})(x-5)}\cdot\left(\frac{5+\sqrt{x+20}}{5+\sqrt{x+20}}\right)$

$\displaystyle = \lim_{x\to5}\frac{25-(x+20)}{5(\sqrt{x+20})(x-5)(5+\sqrt{x+20})} = \lim_{x\to5}\frac{5-x}{5(\sqrt{x+20})(x-5)(5+\sqrt{x+20})}$

$\displaystyle = \lim_{x\to5}\frac{-(x-5)}{5(\sqrt{x+20})(x-5)(5+\sqrt{x+20})}$

$\displaystyle = \lim_{x\to5}\frac{-1}{5(\sqrt{x+20})(5+\sqrt{x+20})} \overset{\text{L.L.}}{=} \frac{-1}{5(5)(5+5)} = \boxed{\frac{-1}{250}}$

Challenge!

**Functions and Limit Practice Problems** Evaluate the following limits:

32. Let $g(x)=2x+1$. Compute $\displaystyle\lim_{x\to1}\frac{x-1}{g(x^2)-3} =$

$$\lim_{x\to 1}\frac{x-1}{(2x^2+1)-3}=\lim_{x\to 1}\frac{x-1}{2x^2-2}=\lim_{x\to 1}\frac{x-1}{2(x-1)(x+1)}=\lim_{x\to 1}\frac{1}{2(x+1)}\overset{\text{DSP}}{=}\boxed{\frac{1}{4}}$$

33. Let $G(u)=u^2+u$. Compute $\displaystyle\lim_{u\to 2}\frac{u^2-2u}{G(u-3)}=\lim_{u\to 2}\frac{u^2-2u}{(u-3)^2+(u-3)}$

$$=\lim_{u\to 2}\frac{u^2-2u}{u^2-6u+9+u-3}=\lim_{u\to 2}\frac{u(u-2)}{u^2-5u+6}=\lim_{u\to 2}\frac{u(u-2)}{(u-3)(u-2)}$$

$$=\lim_{u\to 2}\frac{u}{u-3}\overset{\text{DSP}}{=}\frac{2}{-1}=\boxed{-2}$$

34. Let $h(y)=y^2-3$. Compute $\displaystyle\lim_{x\to -2}\frac{x+2}{h(2x)-h(x+6)}=$

$$\lim_{x\to -2}\frac{x+2}{((2x)^2-3)-((x+6)^2-3)}=\lim_{x\to -2}\frac{x+2}{(4x^2-3)-(x^2+12x+36-3)}$$

$$=\lim_{x\to -2}\frac{x+2}{4x^2-3-x^2-12x-33}=\lim_{x\to -2}\frac{x+2}{3x^2-12x-36}=\lim_{x\to -2}\frac{x+2}{3(x^2-4x-12)}$$

$$=\lim_{x\to -2}\frac{x+2}{3(x-6)(x+2)}=\lim_{x\to -2}\frac{1}{3(x-6)}\overset{\text{DSP}}{=}\boxed{-\frac{1}{24}}$$

35. Let $f(t)=\dfrac{1}{t}$. Compute $\displaystyle\lim_{t\to 4}\frac{f(t-3)-4f(t)}{t-4}=$

$$\lim_{t\to 4}\frac{\left(\dfrac{1}{t-3}-\dfrac{4}{t}\right)}{t-4}=\lim_{t\to 4}\frac{\left(\dfrac{t-4(t-3)}{(t-3)t}\right)}{t-4}=\lim_{t\to 4}\frac{\left(\dfrac{-3t+12}{(t-3)t}\right)}{t-4}$$

$$=\lim_{t\to 4}\left(\frac{-3t+12}{(t-3)t}\right)\cdot\left(\frac{1}{t-4}\right)=\lim_{t\to 4}\frac{-3(t-4)}{(t-3)t(t-4)}$$

$$=\lim_{t\to 4}\frac{-3}{(t-3)t}\overset{\text{DSP}}{=}\frac{-3}{(1)(4)}=\boxed{-\frac{3}{4}}$$

**More Functions**

36. Let $f(x)=\sqrt{x}$, $g(x)=x^2+4$, and $h(x)=\dfrac{1}{x}$. Compute (and simplify, if possible) the following:

(a) $f\circ g(x)=f(g(x))=f\left(x^2+4\right)=\boxed{\sqrt{x^2+4}}$

(b) $g\circ f(x)=g(f(x))=g\left(\sqrt{x}\right)=(\sqrt{x})^2+4=\boxed{x+4}$

(c) $h\circ g\circ f(x)=h(g(f(x))=h(g(\sqrt{x}))=h(x+4)=\boxed{\dfrac{1}{x+4}}$

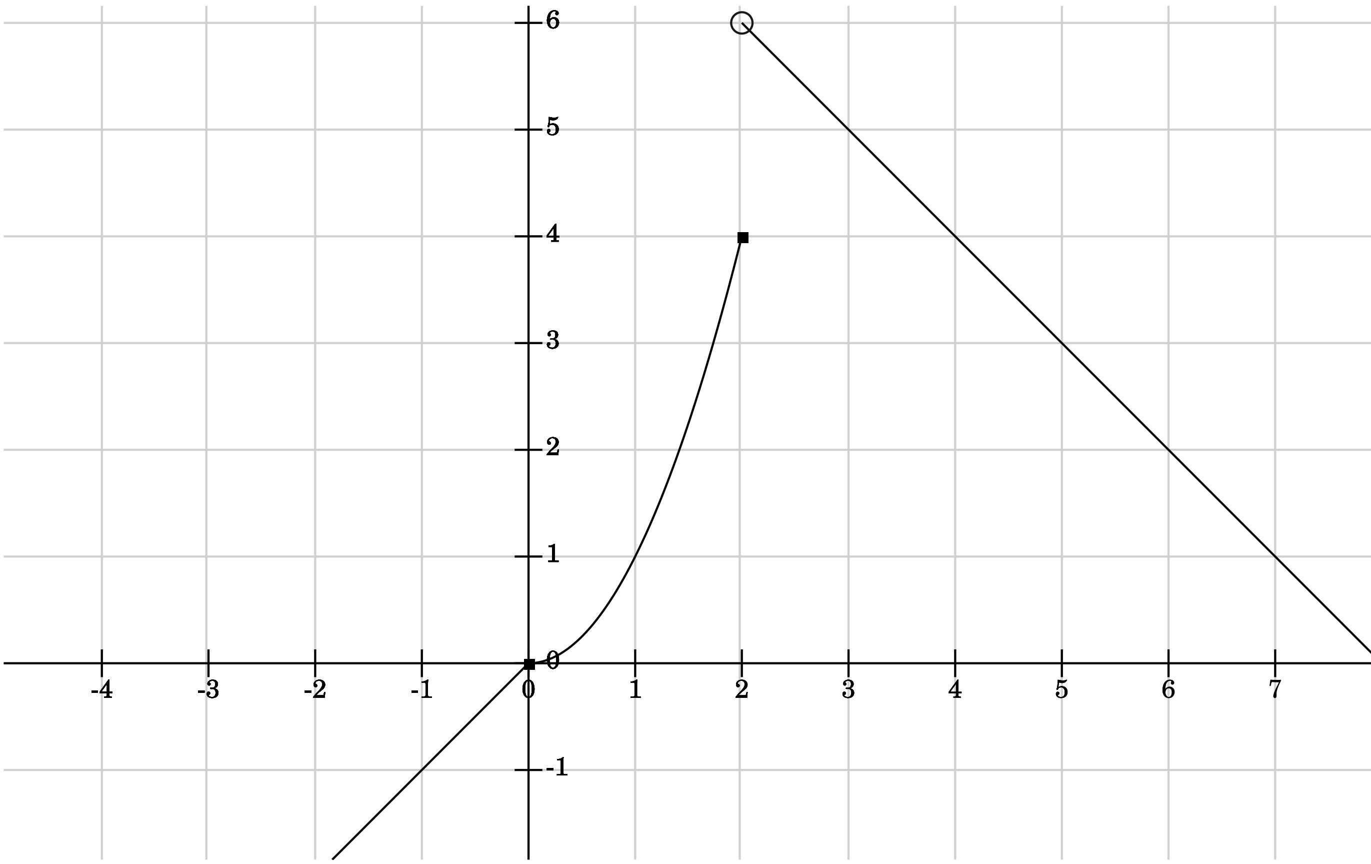
(d) $g\circ g(x)=g(g(x))=g(x^2+4)=(x^2+4)^2+4=x^4+8x^2+16+4=\boxed{x^4+8x^2+20}$

**Piece-wise defined functions**

Consider each of the following piecewise defined functions. Answer the related questions. *Justify* your answers please.

37. Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

Sketch the graph.




Find the numbers at which $f$ is discontinuous.

Evaluate:

$\lim_{x \to 2} f(x) = \boxed{\text{DNE b/c LHL} \neq \text{LHL}}$

RHL: $\lim_{x \to 2^+} f(x) = 6$

LHL: $\lim_{x \to 2^-} f(x) = 4$

$\lim_{x \to 0} f(x) = \boxed{0}$

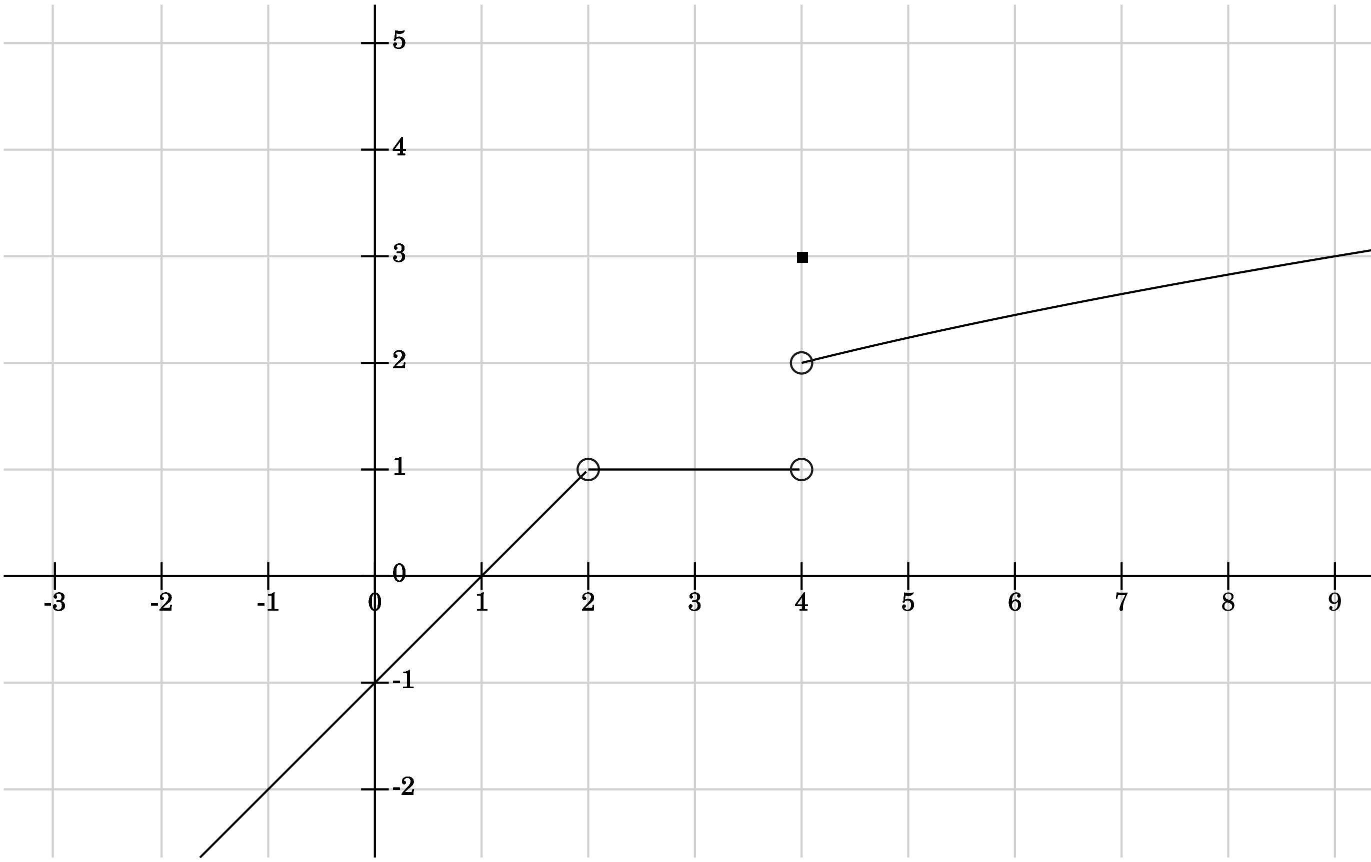
RHL: $\lim_{x \to 0^+} f(x) = 0$

LHL: $\lim_{x \to 0^-} f(x) = 0$

$\boxed{f \text{ is discontinuous at } x = 2 \text{ since } \lim_{x\to 2} f(x) \text{ DNE}}$

38. Let $f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ 1 & \text{if } 2 < x < 4 \\ 3 & \text{if } x = 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

Sketch the graph.




Find the numbers at which $f$ is discontinuous. Evaluate:

$\lim_{x\to 0} f(x) = \boxed{-1}$ b/c RHL=LHL

RHL: $\lim_{x\to 0^+} f(x) = -1$

LHL: $\lim_{x\to 0^-} f(x) = -1$

$\lim_{x\to 2} f(x) = \boxed{1}$ b/c RHL=LHL

RHL: $\lim_{x\to 2^+} f(x) = 1$

LHL: $\lim_{x\to 2^-} f(x) = 1$

$\lim_{x \to 4} f(x) = \boxed{\text{DNE b/c LHL} \neq \text{LHL}}$

RHL: $\lim_{x \to 4^+} f(x) = 2$

LHL: $\lim_{x \to 4^-} f(x) = 1$

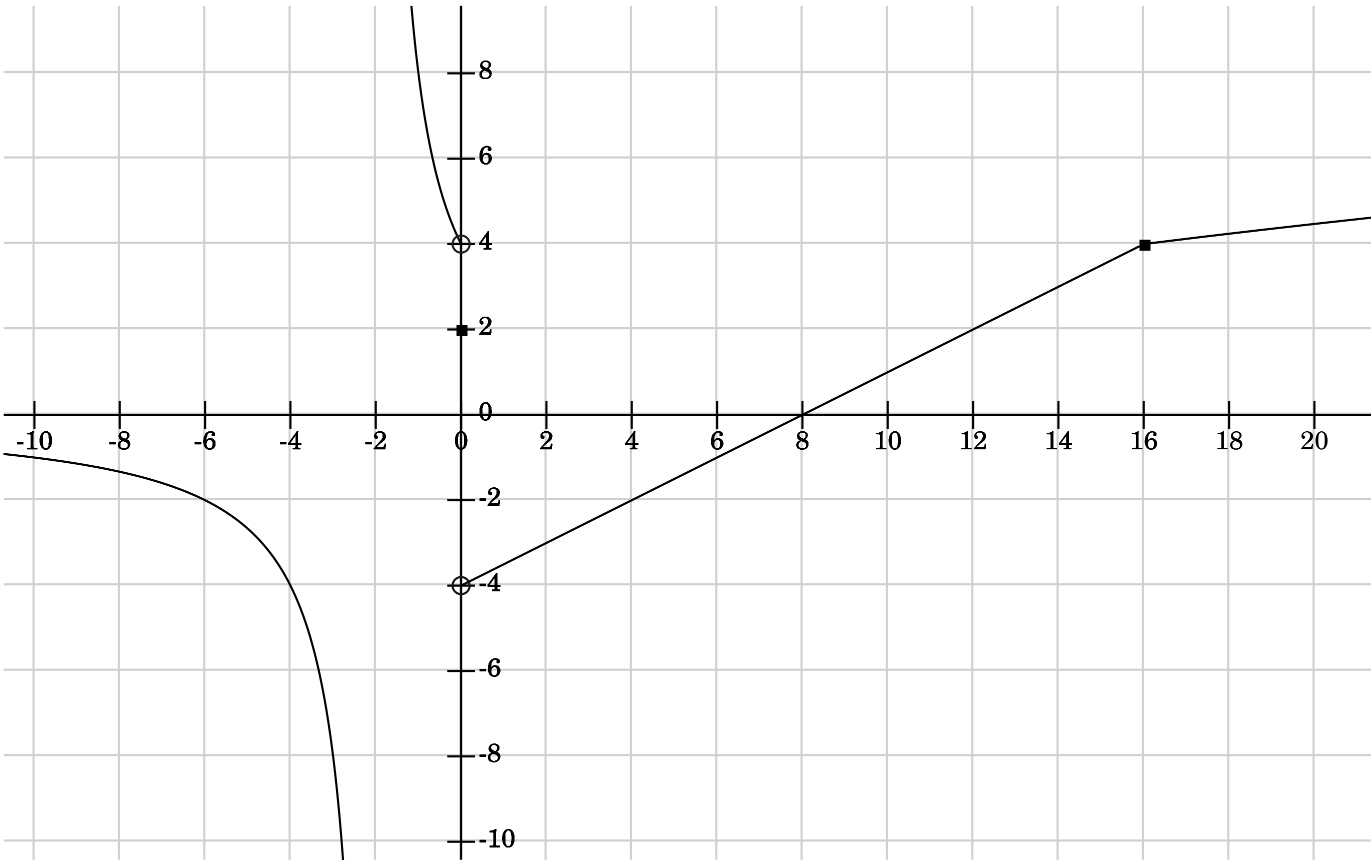
$f(4) = \boxed{3}$

$\boxed{f \text{ is discontinuous at } x = 2 \text{ because } f(2) \text{ is undefined}}$

$\boxed{f \text{ is discontinuous at } x = 4 \text{ because } \lim_{x \to 4} f(x) \ \boxed{\text{DNE b/c LHL} \neq \text{LHL}}}$

39. Let $h(x) = \begin{cases} \dfrac{8}{x+2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } 0 < x \leq 16 \\ \sqrt{x} & \text{if } x > 16 \end{cases}$

Sketch the graph.




Find the numbers at which $h$ is discontinuous. Evaluate:

$\lim_{x \to -2} h(x) = \boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x \to -2^+} h(x) = +\infty$

LHL: $\lim_{x \to -2^-} h(x) = -\infty$

$\lim_{x \to 0} h(x) = \boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x \to 0^+} h(x) = -4$

LHL: $\lim_{x \to 0^-} h(x) = 4$

$\lim_{x \to 16} h(x) = \boxed{4}$ b/c RHL=LHL

RHL: $\lim_{x \to 16^+} h(x) = 4$

LHL: $\lim_{x \to 16^-} h(x) = 4$

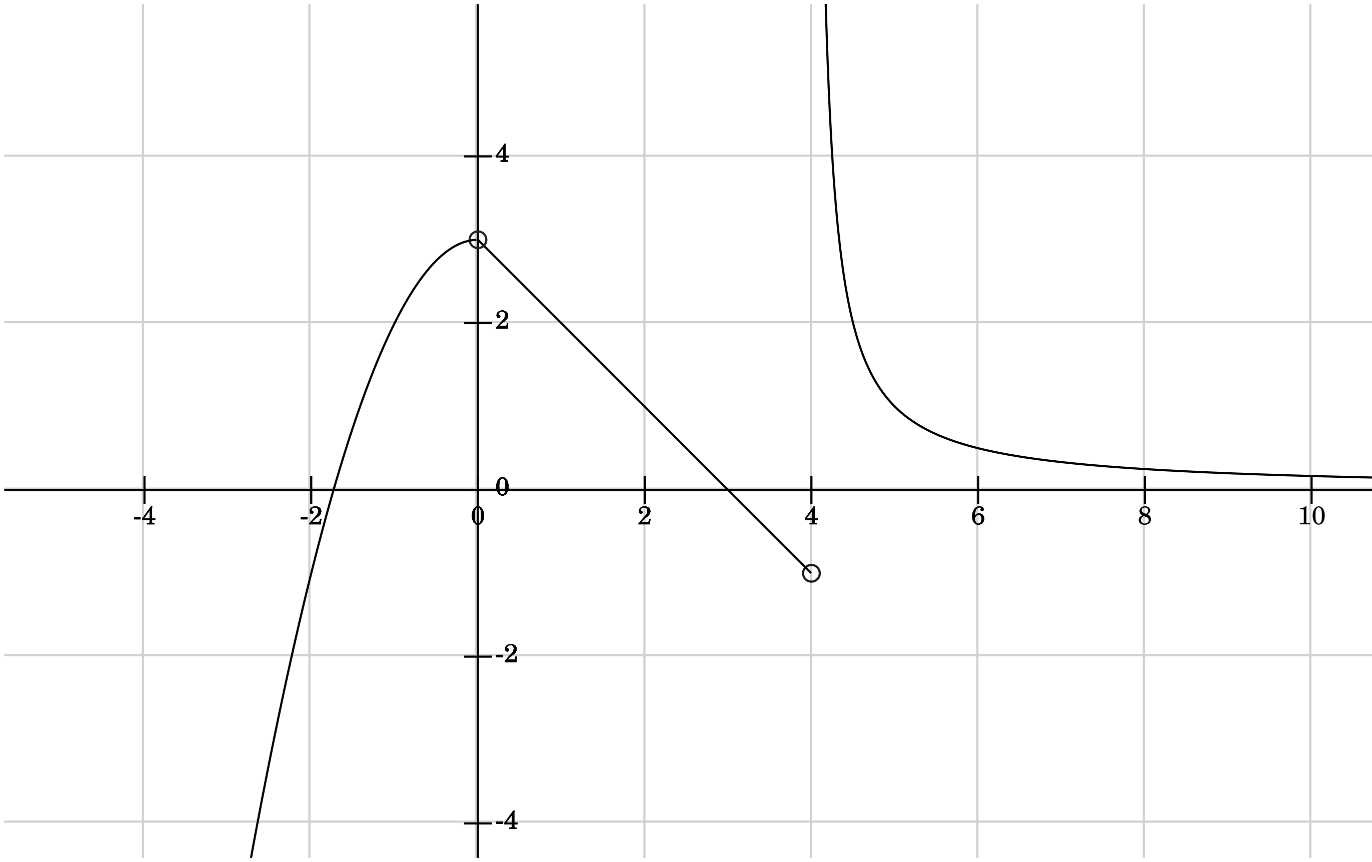
$\boxed{h \text{ is discontinuous at } x = -2 \text{ because } \lim_{x \to -2} h(x) \text{ DNE or because } f(-2) \text{ is undefined}}$

$\boxed{h \text{ is discontinuous at } x = 0 \text{ because } \lim_{x \to 0} h(x) \text{ DNE}}$

Note $h$ is continuous at $x = 16$ because $\lim_{x \to 16} h(x) = h(16)$

40. Let $F(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x > 4 \\ 3 - x & \text{if } 0 < x < 4 \\ 3 - x^2 & \text{if } x < 0 \end{cases}$

Sketch the graph.




Find the numbers at which $F$ is discontinuous. Evaluate:

$\lim_{x\to 0} F(x) = \boxed{3}$ b/c RHL=LHL

RHL: $\lim_{x\to 0^+} F(x) = 3$

LHL: $\lim_{x\to 0^-} F(x) = 3$

$\lim_{x\to 4} F(x) = \boxed{\text{DNE b/c RHL} \neq \text{LHL}}$

RHL: $\lim_{x\to 4^+} F(x) = +\infty$

LHL: $\lim_{x\to 4^-} F(x) = -1$

$\boxed{f \text{ is discontinuous at } x = 0 \text{ because } f(0) \text{ is undefined}}$

$\boxed{f \text{ is discontinuous at } x = 4 \text{ because } f(4) \text{ is undefined or because } \lim_{x\to 4} F(x) \text{ DNE}}$