Math 105 **Practice Exam #1** **Fall 2013** ANSWER KEY

1. Consider the line L given by $x + 3y = 6$.

(a) Sketch this line L.

$x + 3y = 6 \Longrightarrow 3y = -x + 6 \Longrightarrow y = -\frac{1}{3}x + 2$

See graph below.

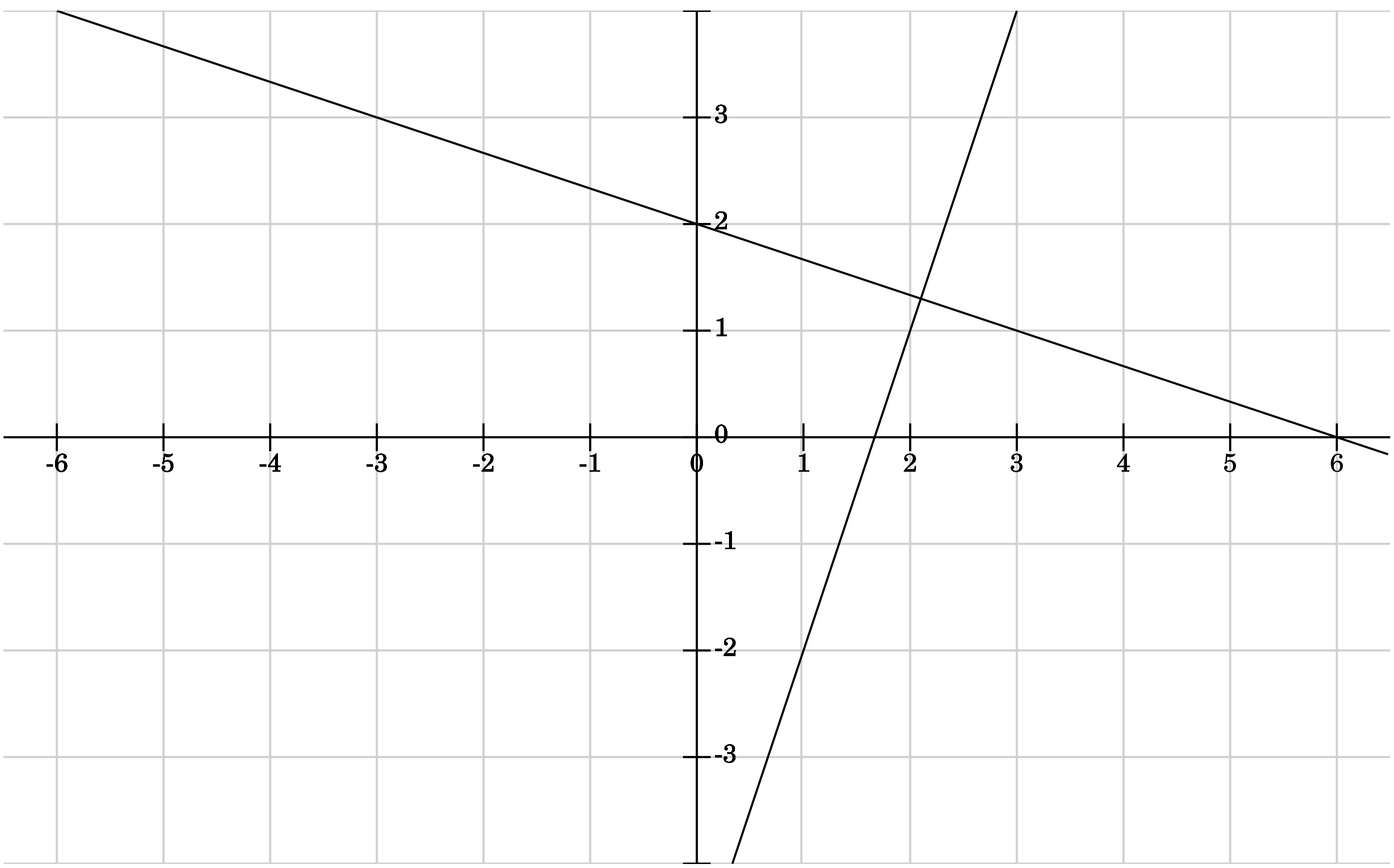
(b) Find the equation of the new line M that is **perpendicular** to the first line L, $x + 3y = 6$, and passes through the point $(1, -2)$.

Line L has slope equal to $-\frac{1}{3}$. Then the perpendicular slope is opposite reciprocal equaling 3.

Then the new line M with slope 3 passing through the point $(1, -2)$ is given by

$y - (-2) = 3(x - 1)$ or $\boxed{y = 3x - 5}$

(c) Sketch this new line M found in (b). See graph below. Both (a) and(c) are on the same graph here.



2. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x\to -7} \dfrac{x^2+5x-14}{x^2-4x+4} \overset{\text{DSP}}{=} \dfrac{0}{81} = \boxed{0}$

(b) $\lim_{x\to 4} \dfrac{x^2-9x+20}{|4-x|} = \boxed{\text{DOES NOT EXIST}}$, RHL$\neq$LHL

RHL: $\lim_{x\to 4^+} \dfrac{x^2-9x+20}{|4-x|} \overset{\frac{0}{0}}{=} \lim_{x\to 4^+} \dfrac{x^2-9x+20}{-(4-x)} = \lim_{x\to 4^+} \dfrac{(x-4)(x-5)}{x-4} = \lim_{x\to 4^+} x-5 \overset{\text{DSP}}{=} \boxed{-1}$

LHL: $\lim_{x\to 4^-} \dfrac{x^2-9x+20}{|4-x|} \overset{\frac{0}{0}}{=} \lim_{x\to 4^-} \dfrac{x^2-9x+20}{4-x} = \lim_{x\to 4^-} \dfrac{(x-4)(x-5)}{-(x-4)} = \lim_{x\to 4^-} -(x-5) \overset{\text{DSP}}{=} \boxed{1}$

Here, recall that $|4-x| = \begin{cases} 4-x & \text{if } \ 4-x \geq 0 \\ -(4-x) & \text{if } \ 4-x < 0 \end{cases} = \begin{cases} 4-x & \text{if } \ x \leq 4 \leftarrow \text{LHL case} \\ x-4 & \text{if } \ x > 4 \leftarrow \text{RHL case} \end{cases}$

Warning: watch the order here on $4-x$. It's not $x-4$.

(c) $\lim_{x\to -6} \dfrac{f(x^2)+5x-8}{[f(x)]^2+5x+14} =$ where $f(x) = x+2$

$$\lim_{x\to -6} \frac{f(x^2)+5x-8}{[f(x)]^2+5x+14} \overset{\frac{0}{0}}{=} \lim_{x\to -6} \frac{x^2+2+5x-8}{[x+2]^2+5x+14} = \lim_{x\to -6} \frac{x^2+5x-6}{x^2+4x+4+5x+14}$$

$$= \lim_{x\to -6} \frac{x^2+5x-6}{x^2+9x+18} = \lim_{x\to -6} \frac{(x+6)(x-1)}{(x+6)(x+3)} = \lim_{x\to -6} \frac{x-1}{x+3} = \frac{-7}{-3} = \boxed{\frac{7}{3}}$$

(d) $\lim_{x\to 2} \dfrac{x^2+5x-14}{x^2-4x+4} \overset{\frac{0}{0}}{=} \lim_{x\to 2} \dfrac{(x+7)(x-2)}{(x-2)(x-2)} = \lim_{x\to 2} \dfrac{x+7}{x-2} \boxed{\text{DOES NOT EXIST}}$, RHL$\neq$LHL

RHL: $\lim_{x\to 2^+} \dfrac{x+7}{x-2} = \dfrac{9}{0^+} = \boxed{+\infty}$

LHL: $\lim_{x\to 2^-} \dfrac{x+7}{x-2} = \dfrac{9}{0^-} = \boxed{-\infty}$

(e) $\lim_{x\to 8} \dfrac{3-\sqrt{x+1}}{x^2-7x-8} \overset{\frac{0}{0}}{=} \lim_{x\to 8} \dfrac{3-\sqrt{x+1}}{x^2-7x-8} \cdot \left(\dfrac{3+\sqrt{x+1}}{3+\sqrt{x+1}}\right) = \lim_{x\to 8} \dfrac{9-(x+1)}{(x^2-7x-8)(3+\sqrt{x+1})}$

$$= \lim_{x\to 8} \frac{8-x}{(x-8)(x+1)(3+\sqrt{x+1})} = \lim_{x\to 8} \frac{-(x-8)}{(x-8)(x+1)(3+\sqrt{x+1})}$$

$$= \lim_{x\to 8} \frac{-1}{(x+1)(3+\sqrt{x+1})} = \frac{-1}{(9)(3+\sqrt{9})} = \frac{-1}{(9)(6)} = \boxed{\frac{-1}{54}}$$

(f) $\lim_{x\to 4} \dfrac{\dfrac{3-x}{x-5} - \dfrac{3}{7-x}}{x^2-x-12} \overset{\frac{0}{0}}{=} \lim_{x\to 4} \dfrac{\left(\dfrac{(3-x)(7-x)-3(x-5)}{(x-5)(7-x)}\right)}{x^2-x-12}$

$$= \lim_{x\to 4} \frac{\left(\dfrac{21 - 10x + x^2 - 3x + 15}{(x-5)(7-x)}\right)}{x^2 - x - 12} = \lim_{x\to 4} \frac{\left(\dfrac{x^2 - 13x + 36}{(x-5)(7-x)}\right)}{x^2 - x - 12}$$

$$= \lim_{x\to 4} \frac{x^2 - 13x + 36}{(x-5)(7-x)} \cdot \left(\frac{1}{x^2 - x - 12}\right) = \lim_{x\to 4} \frac{(x-9)(x-4)}{(x-5)(7-x)} \cdot \left(\frac{1}{(x-4)(x+3)}\right)$$

$$= \lim_{x\to 4} \frac{x-9}{(x-5)(7-x)(x+3)} = \frac{4-9}{(4-5)(7-4)(4+3)} = \frac{-5}{(-1)(3)(7)} = \boxed{\frac{5}{21}}$$

(g) $\displaystyle\lim_{x\to -3} \frac{x-5}{x+3} = \boxed{\text{DNE b/c RHL } \neq \text{ LHL}}$

RHL: $\displaystyle\lim_{x\to -3^+} \frac{x-5}{x+3} = \frac{-8}{0^+} = -\infty$

LHL: $\displaystyle\lim_{x\to -3^-} \frac{x-5}{x+3} = \frac{-8}{0^-} = +\infty$

(h) $\displaystyle\lim_{x\to 8} -\frac{1}{(x-8)^2} = \boxed{-\infty}$ b/c RHL=LHL

RHL: $\displaystyle\lim_{x\to 8^+} -\frac{1}{(x-8)^2} = \frac{-1}{0^+} = -\infty$

LHL: $\displaystyle\lim_{x\to 8^-} -\frac{1}{(x-8)^2} = \frac{-1}{(0^-)^2} = \frac{-1}{0^+} = -\infty$

3. Suppose that $f(x) = \dfrac{x+7}{x-3}$. Compute the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. Simplify your answer until the h in the denominator cancels.

$$\frac{f(x+h) - f(x)}{h} = \frac{\dfrac{(x+h)+7}{(x+h)-3} - \dfrac{x+7}{x-3}}{h} = \frac{\left(\dfrac{(x+h+7)(x-3) - (x+7)(x+h-3)}{(x+h-3)(x-3)}\right)}{h}$$

$$= \frac{x^2 + xh + 7x - 3x - 3h - 21 - (x^2 + xh - 3x + 7x + 7h - 21)}{h(x+h-3)(x-3)}$$

$$= \frac{x^2 + xh + 7x - 3x - 3h - 21 - x^2 - xh + 3x - 7x - 7h + 21}{h(x+h-3)(x-3)}$$

$$= \frac{-3h - 7h}{h(x+h-3)(x-3)} = \frac{-10h}{h(x+h-3)(x-3)} = \boxed{\frac{-10}{(x+h-3)(x-3)}}$$

4. Consider the two functions $f(x) = \dfrac{1+x}{1-x}$ and $g(x) = \dfrac{1}{x}$.

(a) Compute $f \circ g(x)$. Simplify your answer to a single fraction. State the Domain.

$$f \circ g(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1 + \dfrac{1}{x}}{1 - \dfrac{1}{x}} = \frac{\dfrac{x}{x} + \dfrac{1}{x}}{\dfrac{x}{x} - \dfrac{1}{x}} = \frac{\dfrac{x+1}{x}}{\dfrac{x-1}{x}} = \frac{x+1}{x} \cdot \left(\frac{x}{x-1}\right) = \boxed{\frac{x+1}{x-1}}$$

Domain $f \circ g$: $\{x | x \neq 0, 1\}$. Recall, for x to be in the domain of $f \circ g$, it must be in the domain of

g first, and THEN the output $g(x)$ must be in the domain of f.

(b) Compute $g \circ f(x)$. Simplify your answer to a single fraction. State the Domain.

$$g \circ f(x) = g(f(x)) = g\left(\frac{1+x}{1-x}\right) = \frac{1}{\frac{1+x}{1-x}} = \boxed{\frac{1-x}{1+x}}$$

Domain $g \circ f$: $\{x | x \neq -1, 1\}$

(c) Compute $f \circ f(x)$. Simplify your answer to a single fraction. State the Domain.

$$f \circ f(x) = f(f(x)) = f\left(\frac{1+x}{1-x}\right) = \frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}} = \frac{\frac{1-x}{1-x}+\frac{1+x}{1-x}}{\frac{1-x}{1-x}-\frac{1+x}{1-x}} = \frac{\frac{1-x+1+x}{1-x}}{\frac{1-x-(1+x)}{1-x}}$$

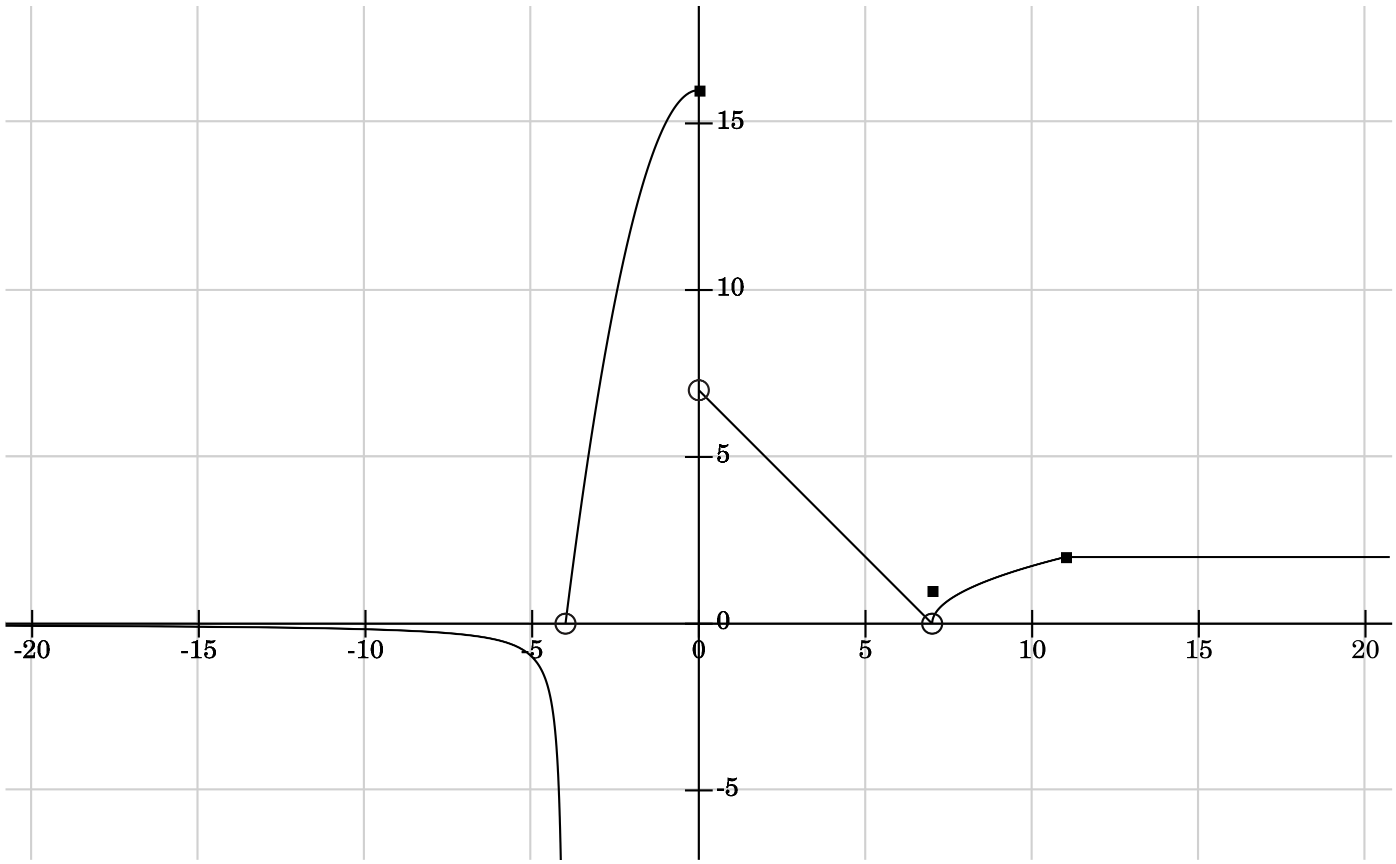
$$= \frac{\frac{1-x+1+x}{1-x}}{\frac{1-x-1-x)}{1-x}} = \frac{\frac{2}{1-x}}{\frac{-2x}{1-x}} = \frac{2}{1-x} \cdot \left(\frac{1-x}{-2x}\right) = \boxed{-\frac{1}{x}}$$

Domain $f \circ f$: $\{x | x \neq 0, 1\}$

5. Consider the function defined by

$$f(x) = \begin{cases} 2 & \text{if } \; x \geq 11 \\ \sqrt{x-7} & \text{if } \; 7 < x < 11 \\ 1 & \text{if } \; x = 7 \\ 7-x & \text{if } \; 0 < x < 7 \\ 16-x^2 & \text{if } \; -4 < x \leq 0 \\ \dfrac{1}{x+4} & \text{if } \; x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.



(b) State the Domain of the function $f(x)$.

Domain= $\boxed{\{x : x \neq -4\}}$

(c) Compute $\lim_{x \to -4} f(x) =$ $\boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\lim_{x \to -4^+} f(x) = \lim_{x \to -4^+} 16 - x^2 = 0$

LHL: $\lim_{x \to -4^-} f(x) = \lim_{x \to -4^-} \frac{1}{x+4} = \frac{1}{0^-} = -\infty$

(d) Compute $\lim_{x \to 0} f(x) =$ $\boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}}$

RHL: $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 7 - x = 7$

LHL: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 16 - x^2 = 16$

(e) Compute $\lim_{x \to 7} f(x) = \boxed{0}$ since RHL=LHL

RHL: $\lim_{x \to 7^+} f(x) = \lim_{x \to 7^+} \sqrt{x-7} = 0$

LHL: $\lim_{x \to 7^-} f(x) = \lim_{x \to 7^-} 7 - x = 0$

(f) Compute $\lim_{x \to 11} f(x) = \boxed{2}$

RHL: $\lim_{x \to 11^+} f(x) = \lim_{x \to 11^+} 2 = 2$

LHL: $\lim_{x \to 11^-} f(x) = \lim_{x \to 11^-} \sqrt{x - 7} = \sqrt{4} = 2$

(g) State the value(s) at which f is discontinuous. Justify your answer(s) using the *definition of continuity* discussed in class.

• f is discontinuous at $x = 7$, because despite the fact that $f(7) = 1$ is defined, and $\lim_{x \to 7} f(x) = 0$, those two values are not equal.

• f is discontinuous at $x = 0$, because despite the fact that $f(0) = 16$ is defined, the $\lim_{x \to 0} f(x)$ DOES NOT EXIST.

• f is discontinuous at $x = -4$ for two reasons, $f(-4)$ is undefined, and the $\lim_{x \to -4} f(x)$ DOES NOT EXIST.

Note that f is continuous at $x = 11$ because $\lim_{x \to 11} f(x) = 2 = f(11)$