

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Simplify your answers.
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [10 Points] Consider the line  $L$  given by  $4x + 2y = 7$ .

- (a) Sketch this line  $L$ .
- (b) Find the equation of the new line  $M$  that is **perpendicular** to the first line  $L$ ,  $4x + 2y = 7$ , and passes through the point  $(4, -1)$ .
- (c) Sketch this new line  $M$  found in (b).

2. [40 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a)  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3} =$       (b)  $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{|x-5|} =$

(c)  $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 3}{x^2 - 2x} =$       (d)  $\lim_{x \rightarrow 3} \frac{\frac{2}{x-1} - \frac{3}{x}}{x-3} =$

(e)  $\lim_{x \rightarrow -3} \frac{G(x^2) - x - 9}{G(x+6) + x^2 + x - 6} =$       where  $G(x) = x - 3$ .      (f)  $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 4x + 4} =$

3. [10 Points] Consider the two functions  $f(x) = \frac{1}{x}$  and  $g(x) = x - 5$ . Compute each of the following. Simplify your answers.

(a)  $f(x+3) =$       (b)  $f(x^2) + 3 =$       (c)  $g(x^2) =$       (d)  $[g(x)]^2 =$   
 (e)  $g \circ f(x) =$       (f)  $f \circ f(x) =$       (g)  $f \circ g(x) =$       (h)  $g \circ g(x) =$

4. [10 Points] For each of the following problems below, sketch any graph for the function  $f$  with the description given.

(a) Sketch a graph of any function  $f$  for which  $\boxed{\lim_{x \rightarrow 2} f(x) \text{ Exists}}$ .

(b) Sketch a graph of any function  $f$  for which  $\boxed{\lim_{x \rightarrow 2} f(x) = 5}$ .

(c) Sketch a graph of any function  $f$  for which  $\boxed{\lim_{x \rightarrow 2} f(x) \text{ Does not Exist}}$ .

(d) Sketch a graph of any function  $f$  for which  $\lim_{x \rightarrow 2} f(x) = -1$  and  $f(2) = 4$ .

(e) Sketch a graph of any function  $f$  for which  $\lim_{x \rightarrow 2} f(x) = -\infty$  and  $f(2)$  is undefined.

5. [10 Points] Suppose that  $f(x) = \sqrt{x-3}$  and  $g(x) = x+5$ .

(a) Compute **and** graph  $f \circ g(x)$ . **Also** state the Domain of  $f \circ g(x)$ .

(b) Compute **and** graph  $g \circ f(x)$ . **Also** state the Domain of  $g \circ f(x)$ .

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \frac{1}{x-3} & \text{if } x > 3 \\ x^2 + 1 & \text{if } 0 < x < 3 \\ -2 & \text{if } x = 0 \\ x + 1 & \text{if } -2 < x < 0 \\ 5 - (x+2)^2 & \text{if } x < -2 \end{cases}$$

(a) Carefully sketch the graph of  $f(x)$ .

(b) State the **Domain** of the function  $f(x)$ .

(c) Compute  $\begin{cases} \lim_{x \rightarrow -2^+} f(x) = \\ \lim_{x \rightarrow -2^-} f(x) = \\ \lim_{x \rightarrow -2} f(x) = \end{cases}$  (d) Compute  $\begin{cases} \lim_{x \rightarrow 0^+} f(x) = \\ \lim_{x \rightarrow 0^-} f(x) = \\ \lim_{x \rightarrow 0} f(x) = \end{cases}$

(e) Compute  $\begin{cases} \lim_{x \rightarrow 3^+} f(x) = \\ \lim_{x \rightarrow 3^-} f(x) = \\ \lim_{x \rightarrow 3} f(x) = \end{cases}$

(f) State the value(s) at which  $f$  is **discontinuous**. Justify your answer(s) using definition of continuity discussed in class.