Math 105 **Answer Key** Exam #1 October 1, 2013

1. [10 Points] Consider the line L given by $4x + 2y = 7$.

(a) Sketch this line L.

$$4x + 2y = 7 \Longrightarrow 2y = -4x + 7 \Longrightarrow y = -2x + \frac{7}{2}$$

See graph below.

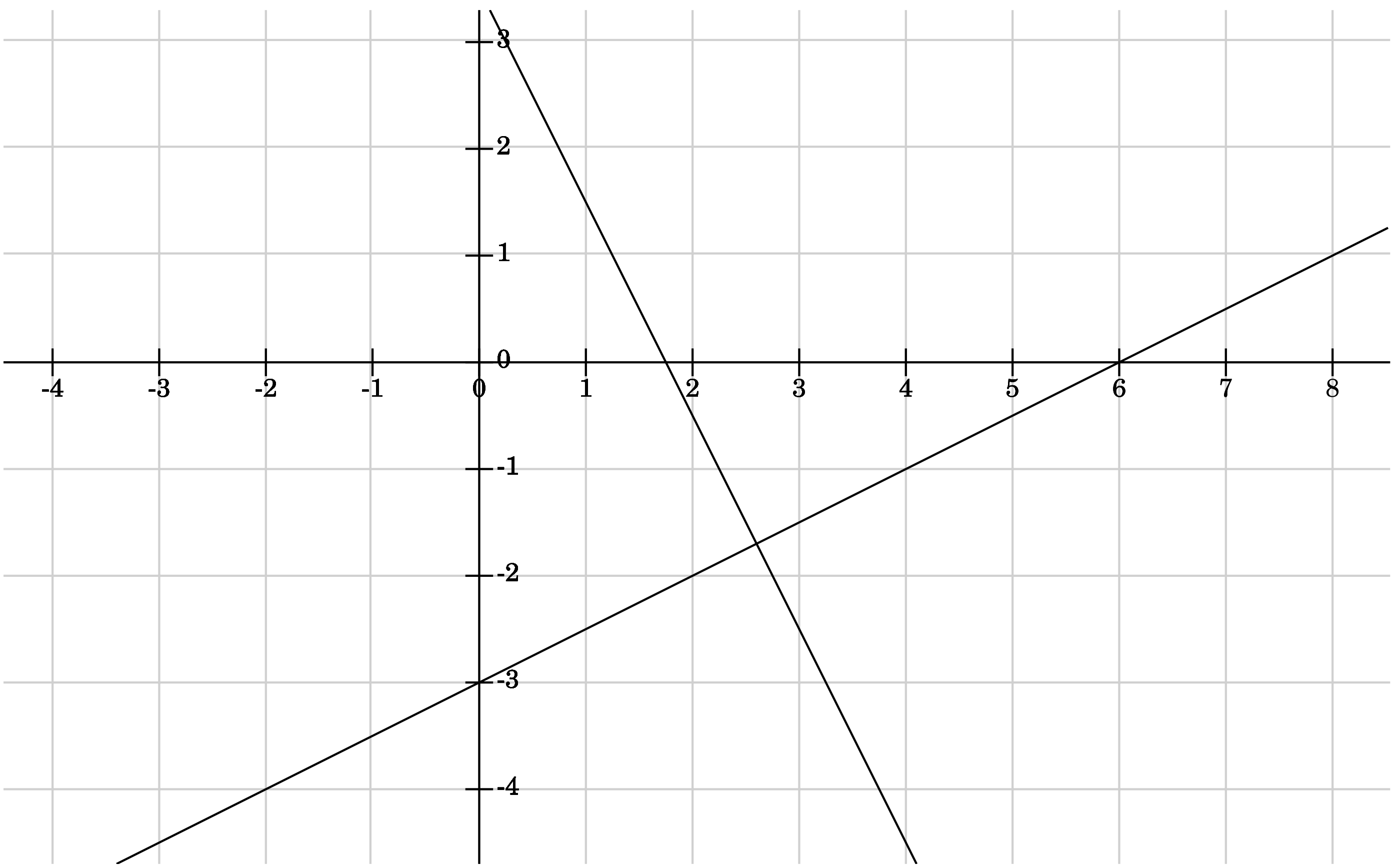
(b) Find the equation of the new line M that is **perpendicular** to the first line L, $4x + 2y = 7$, and passes through the point $(4, -1)$.

Line L has slope equal to $-2\frac{1}{3}$. Then the perpendicular slope is opposite reciprocal equaling 3.

Then the new perp. line M with slope $\frac{1}{2}$ passing through the point $(4, -1)$ is given by

$$y - (-1) = \frac{1}{2}(x - 4) \quad \text{or} \quad \boxed{y = \frac{1}{2}x - 3}$$

(c) Sketch this new line M found in (b).



2. [40 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}\cdot\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right) = \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$

$\displaystyle = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} \overset{\text{L.L.}}{=} \frac{1}{2+2} = \boxed{\frac{1}{4}}$

(b) $\displaystyle \lim_{x\to 5} \frac{x^2-3x-10}{|x-5|} = \boxed{\text{DOES NOT EXIST}}$, RHL$\neq$LHL

RHL: $\displaystyle \lim_{x\to 5^+} \frac{x^2-3x-10}{|x-5|} \overset{\frac{0}{0}}{=} \lim_{x\to 5^+} \frac{x^2-3x-10}{x-5} = \lim_{x\to 5^+} \frac{(x-5)(x+2)}{x-5} = \lim_{x\to 5^+} x+2 \overset{\text{DSP}}{=} \boxed{7}$

LHL: $\displaystyle \lim_{x\to 5^-} \frac{x^2-3x-10}{|x-5|} \overset{\frac{0}{0}}{=} \lim_{x\to 5^-} \frac{x^2-3x-10}{-(x-5)} = \lim_{x\to 5^-} \frac{(x-5)(x+2)}{-(x-5)} = \lim_{x\to 5^-} -(x+2) \overset{\text{DSP}}{=} \boxed{-7}$

Here, recall that $|x-5| = \begin{cases} x-5 & \text{if } x-5\geq 0 \\ -(x-5) & \text{if } x-5<0 \end{cases} = \begin{cases} x-5 & \text{if } x\geq 5 \leftarrow \text{RHL case} \\ -(x-5) & \text{if } x<5 \leftarrow \text{LHL case} \end{cases}$

(c) $\displaystyle \lim_{x\to 1} \frac{x^2-5x+3}{x^2-2x} = \frac{1-5+3}{1-2} \overset{\text{DSP}}{=} \frac{-1}{-1} = \boxed{1}$

(d) $\displaystyle \lim_{x\to 3} \frac{\frac{2}{x-1}-\frac{3}{x}}{x-3} = \lim_{x\to 3} \frac{\frac{2x-3(x-1)}{x(x-1)}}{x-3} = \lim_{x\to 3} \frac{\frac{2x-3x+3}{x(x-1)}}{x-3} = \lim_{x\to 3} \frac{-x+3}{x(x-1)}\left(\frac{1}{x-3}\right)$

$\displaystyle = \lim_{x\to 3} \frac{-(x-3)}{x(x-1)}\left(\frac{1}{x-3}\right) = \lim_{x\to 3} \frac{-1}{x(x-1)} \overset{\text{DSP}}{=} \frac{-1}{2(3)} = \boxed{-\frac{1}{6}}$

(e) $\displaystyle \lim_{x\to -3} \frac{G(x^2)-x-9}{G(x+6)+x^2+x-6} =$ where $G(x) = x-3$.

$\displaystyle \lim_{x\to -3} \frac{G(x^2)-x-9}{G(x+6)+x^2+x-6} = \lim_{x\to -3} \frac{x^2-3-x-9}{(x+6)-3+x^2+x-6}$

$\displaystyle = \lim_{x\to -3} \frac{x^2-x-12}{x^2+2x-3} = \lim_{x\to -3} \frac{(x-4)(x+3)}{(x-1)(x+3)} = \lim_{x\to -3} \frac{x-4}{x-1} \overset{\text{DSP}}{=} \frac{-7}{-4} = \boxed{\frac{7}{4}}$

(f) $\lim_{x\to 2} \dfrac{x^2-7x+10}{x^2-4x+4} = \lim_{x\to 2} \dfrac{(x-5)(x-2)}{(x-2)(x-2)} = \lim_{x\to 2} \dfrac{x-5}{x-2}$ $\boxed{\text{DOES NOT EXIST}}$, RHL$\neq$LHL

RHL: $\lim_{x\to 2^+} \dfrac{x-5}{x-2} = \dfrac{-3}{0^+} = \boxed{-\infty}$

LHL: $\lim_{x\to 2^-} \dfrac{x-5}{x-2} = \dfrac{-3}{0^-} = \boxed{+\infty}$

3. [10 Points] Consider the two functions $f(x) = \dfrac{1}{x}$ and $g(x) = x - 5$. Compute each of the following. Simplify your answers.

(a) $f(x+3) = \boxed{\dfrac{1}{x+3}}$

(b) $f(x^2) + 3 = \boxed{\dfrac{1}{x^2} + 3}$

(c) $g(x^2) = \boxed{x^2 - 5}$

(d) $[g(x)]^2 = \boxed{(x-5)^2}$

(e) $g \circ f(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = \boxed{\dfrac{1}{x} - 5}$

(f) $f \circ f(x) = f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{\left(\dfrac{1}{x}\right)} = \boxed{x}$

(g) $f \circ g(x) = f(g(x)) = f(x-5) = \boxed{\dfrac{1}{x-5}}$

(h) $g \circ g(x) = g(g(x)) = g(x-5) = (x-5) - 5 = \boxed{x - 10}$

4. [10 Points] For each of the following problems below, sketch any graph for the function f with the description given.

(a) Sketch a graph of any function f for which $\boxed{\lim_{x\to 2} f(x) \textbf{ Exists}}$.

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(b) Sketch a graph of any function f for which $\boxed{\lim_{x\to 2} f(x) = 5}$.

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(c) Sketch a graph of any function f for which $\boxed{\lim_{x\to 2} f(x) \quad \textbf{Does not Exist}}$.

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(d) Sketch a graph of any function f for which $\boxed{\lim_{x\to 2} f(x) = -1}$ and $\boxed{f(2) = 4}$.

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(e) Sketch a graph of any function f for which $\boxed{\lim_{x\to 2} f(x) = -\infty}$ and $\boxed{f(2) \text{ is undefined}}$.

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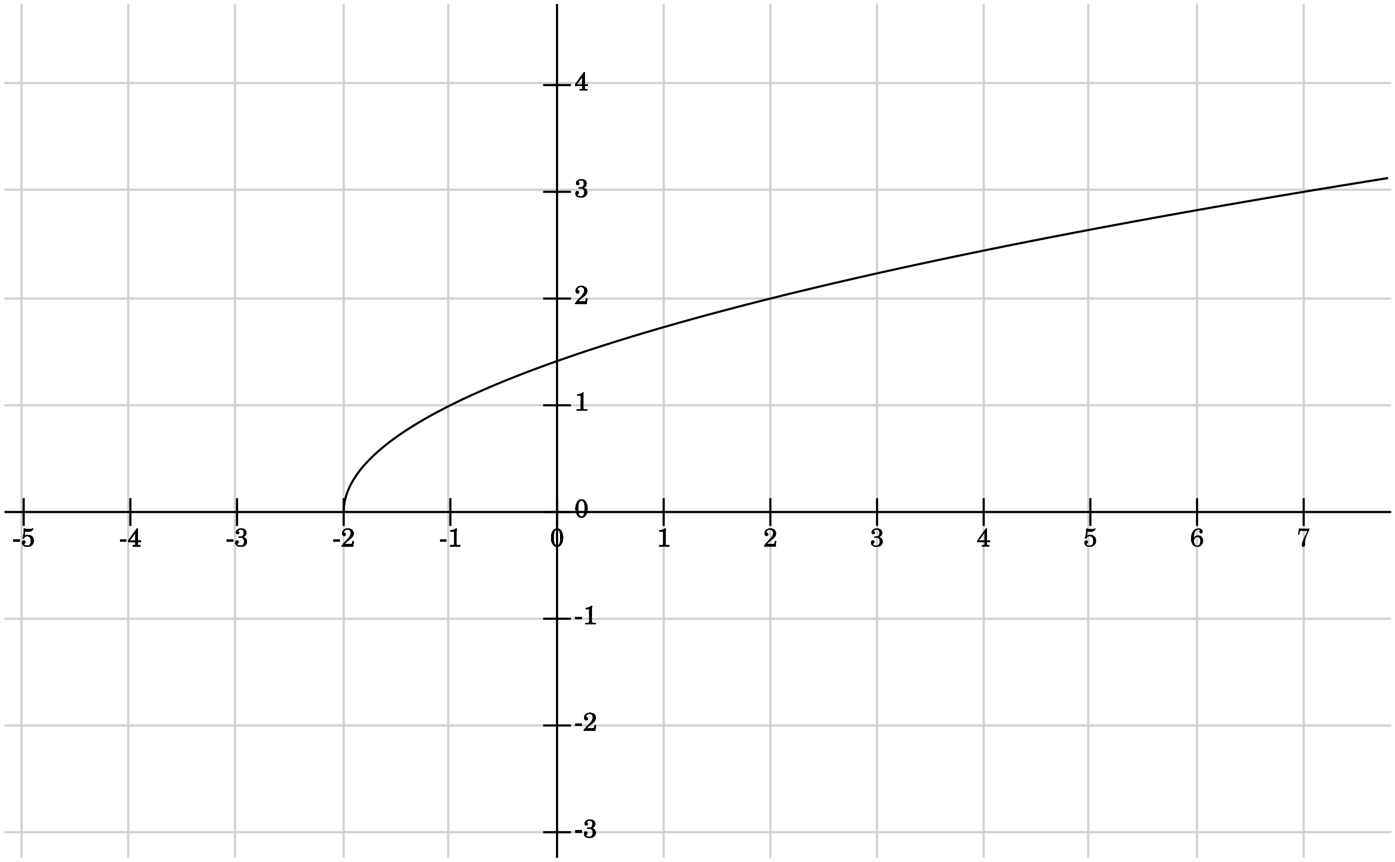
5. [10 Points] Suppose that $f(x) = \sqrt{x-3}$ and $g(x) = x+5$.

(a) Compute **and** graph $f \circ g(x)$. **Also** state the Domain of $f \circ g(x)$.

$f \circ g(x) = f(g(x)) = f(x+5) = \sqrt{x+5-3} = \boxed{\sqrt{x+2}}$

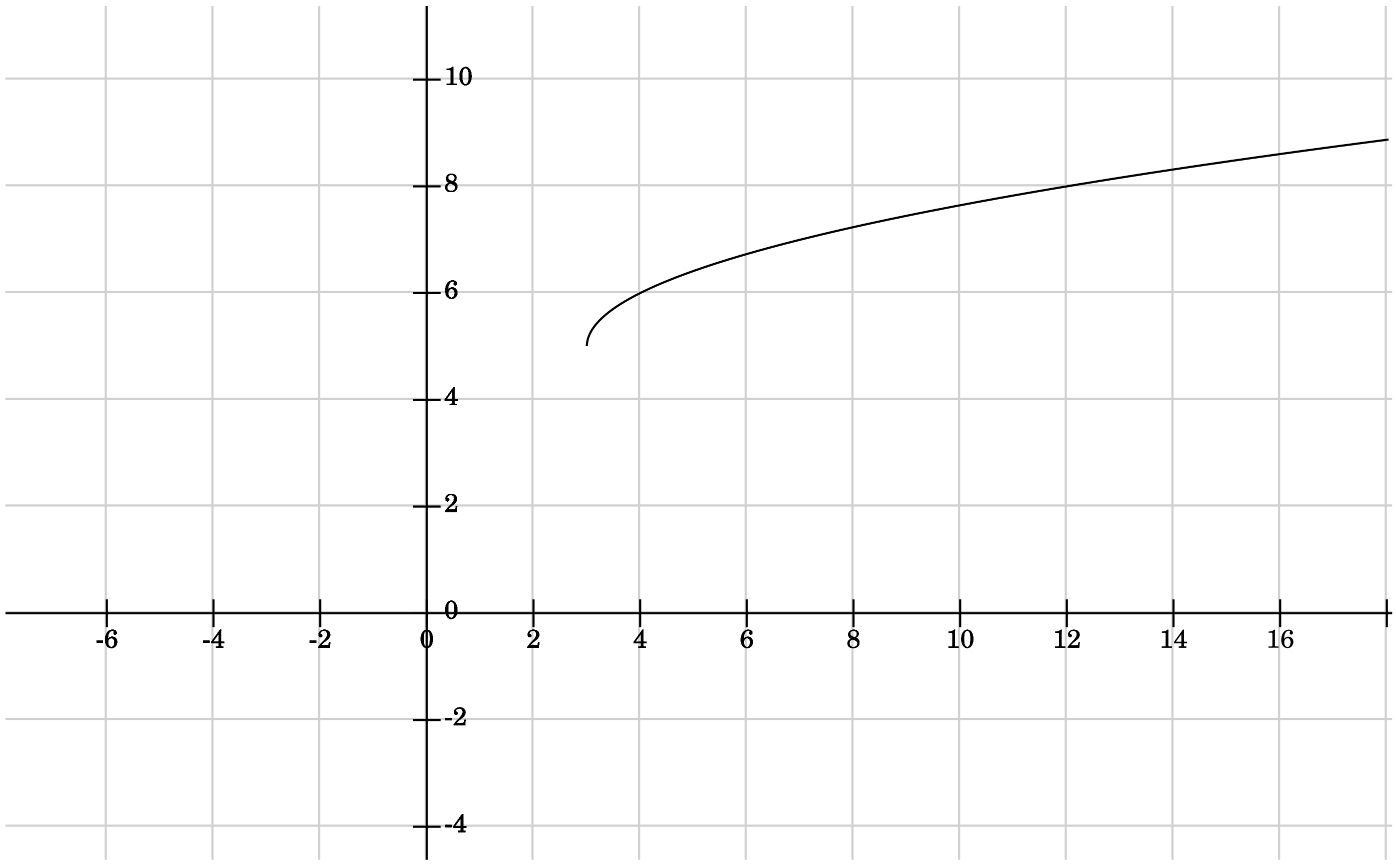
Domain$=\{x|x \geq -2\} = [-2, \infty)$



(b) Compute **and** graph $g \circ f(x)$. **Also** state the Domain of $g \circ f(x)$.

$g \circ f(x) = g(f(x)) = g(\sqrt{x-3}) = \boxed{\sqrt{x-3}+5}$

Domain$=\{x|x \geq 3\} = [3, \infty)$



6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \dfrac{1}{x-3} & \text{if } x > 3 \\ x^2+1 & \text{if } 0 < x < 3 \\ -2 & \text{if } x = 0 \\ x+1 & \text{if } -2 < x < 0 \\ 5-(x+2)^2 & \text{if } x < -2 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

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(b) State the **Domain** of the function $f(x)$.

D=$\{x|x \neq -2, 3\}$

(c) Compute
$$\begin{cases} \lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} x+1 = \boxed{-1} \\ \lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} 5-(x+2)^2 = \boxed{5} \\ \lim_{x\to -2} f(x) = \lim_{x\to -2} f(x) \ \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}} \end{cases}$$

(d) Compute
$$\begin{cases} \lim_{x\to 0^+} f(x) = \boxed{1} \\ \lim_{x\to 0^-} f(x) = \boxed{1} \\ \lim_{x\to 0} f(x) = \boxed{1} \ \text{RHL=LHL} \end{cases}$$

(e) Compute
$$\begin{cases} \lim_{x\to 3^+} f(x) = \boxed{+\infty} \\ \lim_{x\to 3^-} f(x) = \boxed{10} \\ \lim_{x\to 3} f(x) \ \boxed{\text{DOES NOT EXIST since RHL} \neq \text{LHL}} \end{cases}$$

(f) State the value(s) at which f is **discontinuous**. Justify your answer(s) using definition of continuity discussed in class.

- f is discontinuous at $x = -2$, because $f(-2)$ is undefined, OR $\lim_{x\to -2} f(x) = 0$ DOES NOT EXIST.
- f is discontinuous at $x = 0$, because despite the fact that $f(0) = -2$ is defined, and $\lim_{x\to 0} f(x) = 1$, those two values are not equal.
- f is discontinuous at $x = 3$ for two reasons, $f(3)$ is undefined, OR the $\lim_{x\to 3} f(x)$ DOES NOT EXIST.