Extra Example for Cruve Sketching with Vertical and Horizontal Asymptotes

Professor D. Benedetto, Math 105

1. $f(x) = \dfrac{x}{x-2}$

- Domain: $f(x)$ has domain $\{x | x \neq 2\}$

- VA: Vertical asymptote at $x = 2$.

- HA: Horizontal asymptote at $y = 1$ for this f since $\lim\limits_{x \to \pm\infty} f(x) = 1$ because

$$\lim_{x \to \pm\infty} \frac{x}{x-2} \cdot \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \lim_{x \to \pm\infty} \frac{1}{1 - \frac{2}{x}} = 1.$$
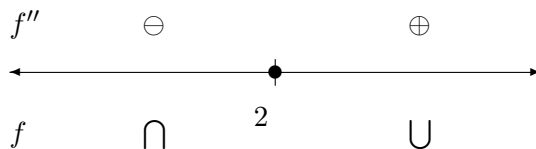
- First Derivative Information:

We compute $f'(x) = \dfrac{-2}{(x-2)^2}$ to find critical numbers. The critical points occur where f' is undefined or zero (never here). The former happens when $x = 2$, which was not in the domain of the original function. As a result, there are no critical numbers. Using sign testing/analysis for f' around $x = 2$,
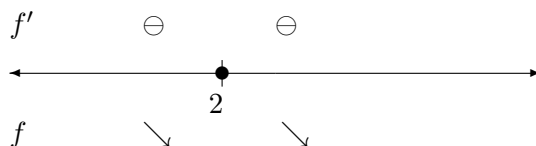
So f is decreasing on $(-\infty, 2)$ and $(2, \infty)$. Moreover, f has no extreme values.
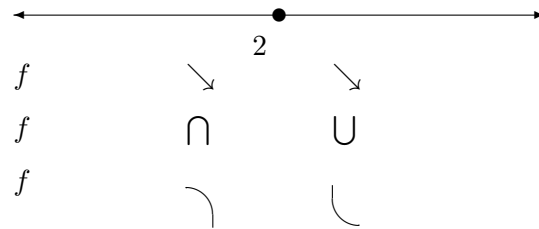
- Second Derivative Information:

Meanwhile, $f'' = \dfrac{4}{(x-2)^3}$. Using sign testing/analysis for f'' around $x = 2$,

So f is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$.

- Piece the first and second derivative information together:

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- Sketch: