Math 105 Practice Final Examination Fall, 2013

1. Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim\limits_{x\to 1} \dfrac{x^2+x-6}{x^2-6x+8}$ (b) $\lim\limits_{x\to 6} \dfrac{x^2-4x-12}{|6-x|}$ (c) $\lim\limits_{x\to 1^+} \dfrac{x^2+x-2}{x^2-2x+1}$

(d) $\lim\limits_{x\to -7} \dfrac{\dfrac{7}{x}-\dfrac{1}{x+6}}{x+7}$ (e) $\lim\limits_{x\to 3^-} \dfrac{x^2-8x+15}{1-8x+g(x+1)}$, where $g(x)=x^2+7$.

2. Compute each of the following derivatives.

(a) $f'(1)$, where $f(x)=\dfrac{x^2+1}{x\sqrt{x}+2x+1}$ Simplify.

(b) $\dfrac{d}{dx}\left(\dfrac{\sqrt{x^7-\dfrac{8}{x^3}}}{x^{\frac{9}{8}}-\dfrac{1}{x^{\frac{8}{9}}}+\dfrac{8}{9}x}\right)$ Do **not** simplify.

(c) $g'''(x)$, where $g(x)=\dfrac{x}{1-2x}$ Simplify.

(d) $\dfrac{dy}{dx}$, if $xy^3+3x^{\frac{7}{4}}=x^2y+7$ Simplify.

(e) $g'(x)$, where $g(x)=\left(\dfrac{3x}{5}-\dfrac{3}{5x}\right)^{-5}\left(5x-\dfrac{\sqrt{x}}{3}\right)^{\frac{3}{5}}$ Do **not** simplify.

(f) $f'(x)$, where $f(x)=x^{\frac{1}{3}}+\dfrac{1}{x^{\frac{1}{3}}}+\dfrac{1}{1+x^{\frac{1}{3}}}+\dfrac{1}{(1+x)^{\frac{1}{3}}}+\dfrac{1}{\left(1+x^{\frac{1}{3}}\right)^{\frac{1}{3}}}$ Do **not** simplify.

3. Let $f(x)=\sqrt{7x-3}$. Calculate $f'(x)$, using the **limit definition** of the derivative. Check your answer using the Chain Rule.

4. Consider the equation $y^2+xy-x^5=8-8x-x^2-y$.

Find the equation of the tangent line to this curve at the point where $(1,0)$.

5. Find the absolute maximum and absolute minimum values of

$$f(x)=x^2\sqrt{5-x} \quad \text{on} \quad [1,5].$$

6. Let $\boldsymbol{f(x) = \dfrac{x-3}{x^4}}$.

Sketch the graph of $y = f(x)$. State the domain for $f(x)$. Clearly indicate horizontal and vertical asymptotes, local minima/maxima, and inflection points on the graph, as well as where the graph is increasing, decreasing, concave up and concave down. Take my word that

$$\boldsymbol{f'(x) = \frac{3(4-x)}{x^5}} \quad \text{and} \quad \boldsymbol{f''(x) = \frac{12(x-5)}{x^6}}.$$

7. A train is travelling east on a straight track at 40 mph. The track is crossed by a road going north and south, and a house is on the road one mile south of the track. Draw the straight line connecting the house to the train. How fast is this distance between the train and the house increasing when the train is 3 miles east of the road?

8. A large box with a square base and top is to be made to hold a fixed volume of 54 cubic feet. The sides cost \$1 per square foot. The top and bottom cost \$2 per square foot. Determine the dimensions that minimize the cost of materials.

(Remember to state the domain of the function you are computing extreme values for.)

9. Consider the function defined by

$$f(x) = \begin{cases} |x-1| & \text{if } x \leq 7 \\ \dfrac{1}{x-7} & \text{if } x > 7 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the domain of the function $f(x)$.

(c) Compute $\begin{cases} \lim\limits_{x \to 7^+} f(x) = \\ \lim\limits_{x \to 7^-} f(x) = \\ \lim\limits_{x \to 7} f(x) = \end{cases}$

(d) State the value(s) of x at which f is discontinuous. Justify your answer(s) using the definition of continuity.

(e) State the value(s) of x where $f(x)$ is *not* differentiable. Justify your answer(s).