Math 105 Final Examination December 16, 2013

1. [40 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to -5}\frac{x^2+2x-15}{x^2-2x-35}$ (b) $\displaystyle\lim_{x\to 2}\frac{g(x^2)+x-3}{[g(x+1)]^2-x+2}$ where $g(x)=x-3$

(c) $\displaystyle\lim_{x\to 5}\frac{x^2-4x-5}{|5-x|}$ (d) $\displaystyle\lim_{x\to 5}\frac{5-x}{\sqrt{x+4}-3}$ (e) $\displaystyle\lim_{x\to 1}\frac{x^2-8x+7}{x^2-2x+1}$ (f) $\displaystyle\lim_{x\to -6}\frac{\frac{x}{x+2}-\frac{x-3}{x}}{x+6}$

2. [40 Points] Compute each of the following derivatives.

(a) $f'(1)$, where $f(x)=\sqrt{\sqrt{x}+\dfrac{3}{\sqrt{x}}}$. Simplify.

(b) $\dfrac{d}{dx}\left(\dfrac{\sqrt{\frac{x^8}{5}-\frac{5}{x^8}}}{x^{\frac{8}{5}}-\frac{1}{x^{\frac{5}{8}}}}\right)$ Do **not** simplify.

(c) $g''(x)$, where $g(x)=\dfrac{x^2}{1-2x^2}$ Simplify.

(d) $\dfrac{dy}{dx}$, if $x^2y^4+5x^{\frac{6}{5}}=xy+8$. Simplify.

(e) $g'(x)$, where $g(x)=\left(\dfrac{3}{x^2}-\dfrac{2}{x^3}\right)^9\left(x^{\frac{5}{6}}-\dfrac{1}{x}\right)$. Do **not** simplify.

(f) $f'(x)$, where $f(x)=x^{\frac{1}{4}}+(1+x)^{\frac{1}{4}}+\left(1+x^{\frac{1}{4}}\right)^{\frac{1}{4}}+\dfrac{1}{x^{\frac{1}{4}}}+\dfrac{1}{1+x^{\frac{1}{4}}}+\dfrac{1}{(1+x)^{\frac{1}{4}}}+\dfrac{1}{\left(1+x^{\frac{1}{4}}\right)^{\frac{1}{4}}}$

Do **not** simplify.

3. [15 Points] Let $f(x)=\dfrac{3-x}{x+7}$.

(a) Compute the derivative of f using the **limit definition** of the derivative.

(b) Compute the derivative of f using the Quotient Rule.

(c) Compute the second derivative $f''(x)$.

4. [10 Points] Consider the equation $y^3+8x=8xy+\sqrt{x}$.

Find the **equation of the tangent line** to this curve at the point $(1,1)$.

5. [15 Points] Find the absolute maximum and absolute minimum values of

$$f(x)=\frac{\sqrt{x-1}}{x} \quad \text{on} \quad [1,10].$$

6. [20 Points] Let $\boldsymbol{f(x) = \dfrac{-x^2+5x-4}{x^2-6x+9} = \dfrac{-x^2+5x-4}{(x-3)^2}}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. **Tip**: $f(7) = -\dfrac{9}{8}$ and $f(9) = -\dfrac{10}{9}$.

Take my word that $\boldsymbol{f'(x) = \dfrac{x-7}{(x-3)^3}}$ and $\boldsymbol{f''(x) = \dfrac{-2x+18}{(x-3)^4}}$.

7. [20 Points] A 10 foot ladder is resting on a vertical wall. The base of the ladder is sliding away from the wall at a rate of 1 foot every second. How fast is the top of the ladder sliding down the wall when the top of the ladder is three feet above the ground?

8. [20 Points] You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

(Don't forget to state the common sense bounds, that is, the domain of the function that you are maximizing or minimizing.)

9. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x > 4 \\ x^2+1 & \text{if } 0 < x < 4 \\ -3 & \text{if } x = 0 \\ x+1 & \text{if } -2 < x < 0 \\ 3-(x+2)^2 & \text{if } x \le -2 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\begin{cases} \lim_{x\to -2^+} f(x) = \\ \lim_{x\to -2^-} f(x) = \\ \lim_{x\to -2} f(x) = \end{cases}$ (d) Compute $\begin{cases} \lim_{x\to 0^+} f(x) = \\ \lim_{x\to 0^-} f(x) = \\ \lim_{x\to 0} f(x) = \end{cases}$ (e) Compute $\begin{cases} \lim_{x\to 4^+} f(x) = \\ \lim_{x\to 4^-} f(x) = \\ \lim_{x\to 4} f(x) = \end{cases}$

(f) State the value(s) at which f is **discontinuous**. Justify your answer(s) using the definition of continuity discussed in class.