

Homework #4

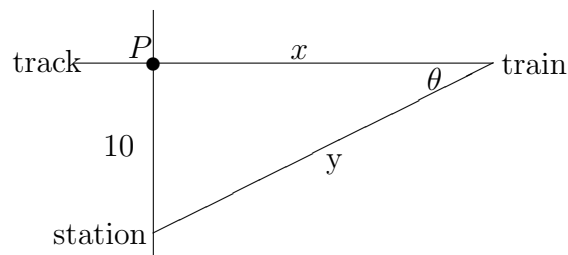
Due Friday, February 7th in Gradescope by 11:59 pm ET

Goal: More Trig Derivatives and Review Related Rates with Trigonometry

FIRST: Read through and understand the following Examples.

Ex: Consider a point P on a train track. Suppose a train depot station is 10 feet directly south from this point P . The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when 2 seconds has passed since the train passed point P .

- Diagram



The picture at arbitrary time t is:

- Variables

Let x = the Distance the train has travelled East past Point P Let y = the Distance between the Train and the Station

Give $\frac{dx}{dt} = 6$ ft/sec. Find $\frac{d\theta}{dt} = ??$ when $x = 6 \left(\frac{\text{ft}}{\text{sec}}\right) \cdot (2\text{seconds}) = 12$ feet

- Equation $\tan \theta = \frac{10}{x} = 10x^{-1}$
- Differentiate $\sec^2 \theta \cdot \frac{d\theta}{dt} = -\frac{10}{x^2} \cdot \frac{dx}{dt}$ **RELATED RATES**
- Extra Solvable Information, **now** you can use the Key Moment information

$$y = \sqrt{(10)^2 + (12)^2} = \sqrt{244} \Rightarrow \sec \theta = \frac{\sqrt{244}}{12} = \frac{\text{Hyp}}{\text{Adj}}$$

- Substitute $\left(\frac{\sqrt{244}}{12}\right)^2 \frac{d\theta}{dt} = -\frac{10}{(12)^2} \cdot 6$

- Solve $\frac{d\theta}{dt} = \frac{-10}{144} \cdot 6 \cdot \frac{144}{244} = -\frac{60}{244} = -\frac{15}{61}$

- Answer in Words: The Angle is *SHRINKING* at $\frac{15}{61}$ Radians per Second *at that moment*.

Next, Complete the following Homework problems.

For #1 – 5, compute the Derivative for each of the following functions. Do **Not** simplify.

1. $f(x) = \cos(3x) \cdot \tan(1 - x^2)$ 2. $y = \sqrt{\cos \sqrt{x}}$ 3. $f(x) = \cos(\sin(\cos x))$

4. $y = \left(\frac{\cos(7x)}{\sin(8x)} \right)^6$ 5. $f(x) = \sin(\tan(\sqrt{9 + x^8}))$

6. If $g(x) = x \sin x$, find $g''(\pi)$

7. Consider $G(x) = 4 \sin^2 x$. Compute $G' \left(\frac{\pi}{6} \right)$. Simplify. Hint: Prep $\sin^2 x = (\sin x)^2$

8. Let $W(x) = \cos^2(2x) + 3 \sec x$. Compute $W' \left(\frac{\pi}{6} \right)$. Simplify.

9. Given $y = 2 \sin x + 3 \cos x$, show that the function satisfies $y'' + y = 0$.

10. Compute $\frac{dy}{dx}$ when $\tan y + \sin x = x^2 - 5y^3$. Hint: Use Implicit Differentiation.

Related Rates with Trigonometry: Give a full detailed and labeled solution for these word problems.

11. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 feet per second, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is 4 ft from the wall?

12. A kite 10 ft above a fixed point P on the ground moves horizontally at a speed of 4 feet per second. At what rate is the angle between the string and the horizontal *decreasing* when 20 ft of string has been let out?

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Andrew, SMUDD **208A**

Wednesday: 1:00-3:00 pm

Thursday: none for Professor

8:00–9:30 pm TA Andrew, SMUDD **208A**

Friday: 12:00–2:00 pm

- Related Rates with Trig functions keeps the same Related Rates Structure but uses different Equations. Label your steps.