Worksheet 9 Answer Key

1. $g(x) = \int_{\boxed{x}}^{7} \sqrt{e^t+3}\,dt = -\int_7^x \sqrt{e^t+3}\,dt$

$g'(x) = -\sqrt{e^x+3}$ FTC Part I

$$g''(x) = \boxed{-\frac{1}{2\sqrt{e^x+3}} \cdot e^x}$$

2. $f(x) = \sqrt{\cos(x^2+e^x)} + \cos\sqrt{x^2+e^x} + e^{\sqrt{x^2+\cos x}}$

$$f'(x) = \frac{1}{2\sqrt{\cos(x^2+e^x)}} \cdot \left(-\sin(x^2+e^x)\right) \cdot (2x+e^x) - \sin\sqrt{x^2+e^x} \cdot \frac{1}{2\sqrt{x^2+e^x}} \cdot (2x+e^x) \ldots$$

Continued $\ldots + e^{\sqrt{x^2+\cos x}} \cdot \dfrac{1}{2\sqrt{x^2+\cos x}} \cdot (2x - \sin x)$

3. $\frac{d}{dx}\left(e^{\sin y}\right) = \frac{d}{dx}(2 - xy)$ Implicit Differentiation

$$e^{\sin y} \cdot \cos y \cdot \frac{dy}{dx} = -\left(x \cdot \frac{dy}{dx} + y \cdot (1)\right)$$

$$e^{\sin y} \cos y \cdot \frac{dy}{dx} = -x\frac{dy}{dx} - y$$

$$e^{\sin y} \cos y \frac{dy}{dx} + x\frac{dy}{dx} = -y$$

$$\left(e^{\sin y} \cdot \cos y + x\right)\frac{dy}{dx} = -y$$ Isolate and Solve

$$\frac{dy}{dx} = \boxed{\frac{-y}{e^{\sin y} \cdot \cos y + x}}$$

4. $f(x) = \dfrac{1+e^{-2x}}{1-e^{7x}}$ Quotient Rule

$$f'(x) = \frac{(1-e^{7x}) \cdot e^{-2x} \cdot (-2) - (1+e^{-2x})(-e^{7x}) \cdot 7}{(1-e^{7x})^2}$$ FOIL

$$= \frac{-2e^{-2x} + 2e^{5x} + 7e^{7x} + 7e^{5x}}{(1-e^{7x})^2}$$

$$= \boxed{\frac{7e^{7x} + 9e^{5x} - 2e^{-2x}}{(1-e^{7x})^2}}$$

5. $\int e^x(1+e^x)^2\,dx = \int u^2\,du = \frac{u^3}{3}+C = \boxed{\frac{(1+e^x)^3}{3}+C}$

$\boxed{\begin{array}{l} u = 1+e^x \\ du = e^x\,dx \end{array}}$

split

6. $\int \frac{(1+e^x)^2}{e^x}\,dx = \int \frac{1+2e^x+e^{2x}}{e^x}\,dx = \int \frac{1}{e^x} + \frac{2\cancel{e^x}}{\cancel{e^x}} + \frac{e^{2x}}{e^x}\,dx$

$= \int e^{-x} + 2 + e^x\,dx$

K-rule

$= \frac{e^{-x}}{-1} + 2x + e^x + C$

$= \boxed{-\frac{1}{e^x} + 2x + e^x + C}$

~~Try u-sub~~
~~$u = 1+e^x$~~
~~$du = e^x\,dx$~~ NO MATCH!

FOIL + Algebra

constant

K-rule: $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$

cancel

7. $\int (e^x+e^{-x})(e^x-e^{-x})\,dx = \int e^{2x} \overset{-1}{\cancel{-e^0}} \overset{+1}{\cancel{+e^0}} - e^{-2x}\,dx$

Method 1: Algebra FOIL

$= \frac{e^{2x}}{2} \cancel{-} \frac{e^{-2x}}{\cancel{-}2} + C$

$= \boxed{\frac{e^{2x}}{2} + \frac{e^{-2x}}{2} + C}$

Equivalent if FOIL and absorb cross term constant into +C piece.

OR//

Method 2: u-sub

$\boxed{\begin{array}{l} u = e^x + e^{-x} \\ du = e^x - e^{-x}\,dx \end{array}}$

$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(e^x+e^{-x})^2}{2} + C}$

$= \frac{e^{2x} + 2e^0 + e^{-2x}}{2} + C$

$= \frac{e^{2x}+e^{-2x}}{2} + C$ Match

8. $\int (e^{4x}+e^{-9x})^2\,dx = \int (e^{4x}+e^{-9x})(e^{4x}+e^{-9x})\,dx$

FOIL Algebra

$= \int e^{8x} + e^{-5x} + e^{-5x} + e^{-18x}\,dx$

$= \int e^{8x} + 2e^{-5x} + e^{-18x}\,dx$

$= \boxed{\frac{e^{8x}}{8} + \frac{2e^{-5x}}{-5} + \frac{e^{-18x}}{-18} + C}$ OR// $= \boxed{\frac{e^{8x}}{8} - \frac{2}{5e^{5x}} - \frac{1}{18e^{18x}} + C}$

9. $\displaystyle\int \frac{\sqrt{1+e^{-3x}}}{e^{3x}}\,dx = -\frac{1}{3}\int \sqrt{u}\,du = -\frac{1}{3}\,\frac{u^{3/2}}{3/2} + C = \boxed{-\frac{2}{9}\left(1+e^{-3x}\right)^{3/2} + C}$

$u^{1/2}$ $\quad \frac{2}{3}$

$$\boxed{\begin{aligned} u &= 1+e^{-3x} \\ du &= -3e^{-3x}\,dx \\ -\frac{1}{3}\,du &= \frac{1}{e^{3x}}\,dx \end{aligned}}$$

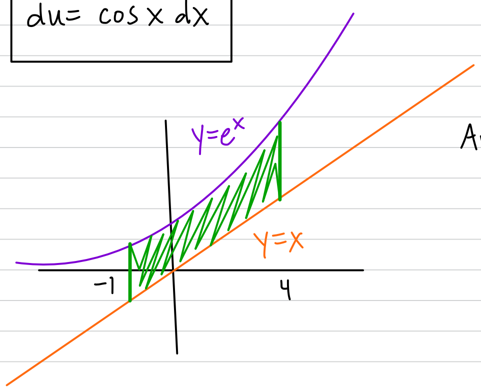
10. $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx = -\int e^u\,du = -e^u + C = -e^{\frac{1}{x}} + C$

$$\boxed{\begin{aligned} u &= \frac{1}{x} \\ du &= -\frac{1}{x^2}\,dx \\ -du &= \frac{1}{x^2}\,dx \end{aligned}}$$

11. $\displaystyle\int \cos x \cdot e^{5+\sin x}\,dx = \int e^u\,du = e^u + C = \boxed{e^{5+\sin x} + C}$

$$\boxed{\begin{aligned} u &= 5+\sin x \\ du &= \cos x\,dx \end{aligned}}$$

12.



$$\text{Area} = \int_{-1}^{4} \text{TOP} - \text{BOTTOM}\,dx$$

$$= \int_{-1}^{4} e^x - x\,dx$$

$$= e^x - \frac{x^2}{2}\Big|_{-1}^{4}$$

$$= e^4 - \frac{\cancel{16}^{\,8}}{\cancel{2}} - \left(e^{-1} - \frac{1}{2}\right)$$

$$= e^4 - \underbrace{8}_{-\frac{16}{2}} - \frac{1}{e} + \frac{1}{2} = \boxed{e^4 - \frac{1}{e} - \frac{15}{2}}$$

13. $f(x) = \ln\left(\cos x + \sqrt{x}\right)$

$$f'(x) = \boxed{\frac{1}{\cos x + \sqrt{x}} \cdot \left(-\sin x + \frac{1}{2\sqrt{x}}\right)}$$

K-rule

14. $$\int_0^{\ln 5} \frac{1}{e^{2x}}\,dx = \int_0^{\ln 5} e^{-2x}\,dx = \frac{e^{-2x}}{-2}\Bigg|_0^{\ln 5} = -\frac{1}{2}e^{-2\ln 5} + \frac{1}{2}\cancel{e^{0}}^{\,1}$$

$$= -\frac{1}{2}\cdot\frac{1}{e^{2\ln 5}} + \frac{1}{2} = -\frac{1}{2}\cdot\frac{1}{\underbrace{\cancel{e}^{\cancel{\ln}(5^2)}}_{25}} + \frac{1}{2}$$

$$= -\frac{1}{50} + \frac{1}{2} = -\frac{1}{50} + \frac{25}{50} = \frac{24}{50} = \frac{12}{25}$$

15. $$\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx = \int_1^2 \frac{1}{u^2}\,du = \int_1^2 u^{-2}\,du = \frac{u^{-1}}{-1}\Bigg|_1^2 = -\frac{1}{u}\Bigg|_1^2$$

$$= -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x = e &\Rightarrow u = \cancel{\ln}\,\cancel{e} = 1 \\ x = e^2 &\Rightarrow u = \cancel{\ln}\,\cancel{e}^2 = 2 \end{aligned}}$$

16. $$f(x) = \int f'(x)\,dx = \int \frac{e^{\sqrt{\tan x}}\cdot \sec^2 x}{\sqrt{\tan x}}\,dx = 2\int e^u\,du = 2e^u + C$$

$$= 2e^{\sqrt{\tan x}} + \cancel{C}^{\,1-2e}$$

$$\boxed{\begin{aligned} u &= \sqrt{\tan x} \\ du &= \frac{1}{2\sqrt{\tan x}}\cdot \sec^2 x\,dx \\ 2\,du &= \frac{1}{\sqrt{\tan x}}\cdot \sec^2 x\,dx \end{aligned}}$$

Test: $f\left(\frac{\pi}{4}\right) = 2e^{\sqrt{\tan \pi/4}} + C \overset{\text{Set}}{=} 1$

$$2e + C = 1$$

$$\Rightarrow C = 1 - 2e$$

Finally, $f(x) = \boxed{2e^{\sqrt{\tan x}} + (1 - 2e)}$