Worksheet 7 Answer Key

1(a) $$\int_1^5 7-x-x^2\,dx = 7x-\frac{x^2}{2}-\frac{x^3}{3}\Big|_1^5 = 35-\frac{25}{2}-\frac{125}{3}-\left(7-\frac{1}{2}-\frac{1}{3}\right)$$

$$= 35-\frac{25}{2}-\frac{125}{3}-7+\frac{1}{2}+\frac{1}{3} = 28-\cancel{\frac{24}{2}}^{-12}-\frac{124}{3} = 16-\frac{124}{3} = \frac{48}{3}-\frac{124}{3} = -\frac{76}{3}$$

(b) $$\int_1^5 7-x-x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(1+\frac{4i}{n}\right)\cdot\frac{4}{n}$$

Here $f(x)=7-x-x^2$

$a=1 \quad b=5$

$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

$x_i = a+i\Delta x$

$= 1+i\left(\frac{4}{n}\right)$

$= 1+\frac{4i}{n}$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n} f\left(1+\frac{4i}{n}\right)$$

$$-\left(1+\frac{8i}{n}+\frac{16i^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n} 7-\left(1+\frac{4i}{n}\right)-\left(1+\frac{4i}{n}\right)^2$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n} 7-1-\frac{4i}{n}-1-\frac{8i}{n}-\frac{16i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n} 5-\frac{12i}{n}-\frac{16i^2}{n^2}$$

$$= \lim_{n\to\infty}\frac{4}{n}\left[\sum_{i=1}^{n} 5-\sum_{i=1}^{n}\frac{12i}{n}-\sum_{i=1}^{n}\frac{16i^2}{n^2}\right]$$

$$= \lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n} 5-\frac{4}{n}\sum_{i=1}^{n}\frac{12i}{n}-\frac{4}{n}\sum_{i=1}^{n}\frac{16}{n^2}i^2$$

$$= \lim_{n\to\infty}\frac{20}{n}\cancel{\sum_{i=1}^{n}1}^{\,n}-\frac{48}{n^2}\cancel{\sum_{i=1}^{n}i}-\frac{64}{n^3}\cancel{\sum_{i=1}^{n}i^2}$$

$$= \lim_{n\to\infty}\frac{20}{\cancel{n}}\cdot\cancel{n}-\frac{48}{n^2}\left(\frac{n(n+1)}{2}\right)-\frac{64}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} 20-\frac{48}{2}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)-\frac{64}{6}\left(\frac{\cancel{n}}{\cancel{n}}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$$

$$= \lim_{n\to\infty} 20-24(1)\left(1+\cancel{\frac{1}{n}}^{0}\right)-\frac{64}{6}(1)\left(1+\cancel{\frac{1}{n}}^{0}\right)\left(2+\cancel{\frac{1}{n}}^{0}\right)$$

$$= 20-24(1)(1)-\frac{64}{6}(1)(1)(2)$$

$$= 20-24-\frac{64}{3}$$

$$= -4-\frac{64}{3} = -\frac{12}{3}-\frac{64}{3} = \frac{-76}{3}$$

2. $$g(x) = \int_x^2 \frac{\cos t}{5+\cos t}\,dt = -\int_2^x \frac{\cos t}{5+\cos t}\,dt$$

$$g'(x) = \frac{d}{dx}\left(-\int_2^x \frac{\cos t}{5+\cos t}\,dt\right) = -\frac{\cos x}{5+\cos x}$$

3. $$\int_0^4 \frac{1-x}{\sqrt{x}}\,dx = \int_0^4 \frac{1}{\sqrt{x}} - \frac{x}{\sqrt{x}}\,dx = \int_0^4 x^{-1/2} - x^{1/2}\,dx = \frac{x^{1/2}}{1/2} - \frac{x^{3/2}}{3/2}\Bigg|_0^4$$

$$= 2\sqrt{x} - \frac{2}{3}x^{3/2}\Bigg|_0^4 = 2\overset{2}{\cancel{\sqrt{4}}} - \frac{2}{3}\cdot 4^{\frac{3}{2}} - (0-0)$$

$(\sqrt{4})^3$

$$= 4 - \frac{16}{3} = \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}$$

4. $$\int x^4(2-3x^5)^6\,dx = -\frac{1}{15}\int u^6\,du = -\frac{1}{15}\cdot\frac{u^7}{7} + C = -\frac{(2-3x^5)^7}{105} + C$$

$$\boxed{\begin{aligned} u &= 2-3x^5 \\ du &= -15x^4\,dx \\ -\frac{1}{15}\,du &= x^4\,dx \end{aligned}}$$

5. $$\int_9^{64} \frac{5}{\sqrt{x}\sqrt{1+\sqrt{x}}}\,dx = 5\cdot 2\int_4^9 \frac{1}{\sqrt{u}}\,du = \frac{10\,u^{1/2}}{1/2}\Bigg|_4^9 = 20\sqrt{u}\Bigg|_4^9$$

$u^{-1/2}$

$$= 20\left(\overset{3}{\cancel{\sqrt{9}}} - \overset{2}{\cancel{\sqrt{4}}}\right)$$

$$= 20\cdot 1 = 20$$

$$\boxed{\begin{aligned} u &= 1+\sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \\ 2\,du &= \frac{1}{\sqrt{x}}\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x = 9 &\Rightarrow u = 1+\overset{3}{\cancel{\sqrt{9}}} = 4 \\ x = 64 &\Rightarrow u = 1+\overset{8}{\cancel{\sqrt{64}}} = 9 \end{aligned}}$$

6. $$\int_0^{\frac{\pi}{6}} \frac{\cos x}{(1+6\sin x)^2}\,dx = \frac{1}{6}\int_1^4 \frac{1}{u^2}\,du = \frac{1}{6}\int_1^4 u^{-2}\,du = \frac{1}{6}\left(\frac{u^{-1}}{-1}\right)\Bigg|_1^4 = -\frac{1}{6u}\Bigg|_1^4$$

$$\boxed{\begin{aligned} u &= 1+6\sin x \\ du &= 6\cos x\,dx \\ \tfrac{1}{6}\,du &= \cos x\,dx \end{aligned}}$$

$$\boxed{\begin{aligned} x=0 &\Rightarrow u = 1+6\sin \overset{0}{\cancel{0}} = 1 \\ x=\tfrac{\pi}{6} &\Rightarrow u = 1+\overset{3}{\cancel{6}}\,\overset{\frac{1}{2}}{\cancel{\sin\tfrac{\pi}{6}}} \\ &= 1+3 = 4 \end{aligned}}$$

$$= -\frac{1}{24} + \frac{1}{6} = -\frac{1}{24} + \frac{4}{24} = \frac{3}{24} = \frac{1}{8}$$

7. $$\int \frac{5}{x^2\left(5+\frac{3}{x}\right)^{\frac{3}{5}}}\,dx = 5\cdot\left(-\frac{1}{3}\right)\int \frac{1}{u^{\frac{3}{5}}}\,du \overset{\text{prep}}{=} -\frac{5}{3}\int u^{-\frac{3}{5}}\,du$$

$$= -\frac{5}{3}\,\frac{u^{\frac{2}{5}}}{\frac{2}{5}} + C \qquad \frac{5}{2}\cdot$$

$$= \frac{-25}{6}\left(5+\frac{3}{x}\right)^{\frac{2}{5}} + C$$

$$\boxed{\begin{aligned} u &= 5+\frac{3}{x} \to 3x^{-1} \\ du &= -3x^{-2}\,dx \\ -\frac{1}{3}\,du &= \frac{1}{x^2}\,dx \end{aligned}}$$

Challenge:

8. $$\int x(x-2)^{\frac{3}{4}}\,dx = \int (u+2)\,u^{3/4}\,du = \int u^{7/4} + 2u^{3/4}\,du$$

$$\boxed{\begin{aligned} u = x-2 &\Rightarrow x = u+2 \\ du = dx & \end{aligned}}$$

$$= \frac{u^{11/4}}{11/4} + \frac{2u^{7/4}}{7/4} + C$$

$$= \boxed{\frac{4}{11}(x-2)^{11/4} + \frac{8}{7}(x-2)^{7/4} + C}$$

9. $$f(x) = \int \frac{\sec^2 x}{\sqrt{3+\tan x}}\,dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\boxed{\begin{aligned} u &= 3+\tan x \\ du &= \sec^2 x\,dx \end{aligned}}$$

$$= 2\sqrt{u} + C = 2\sqrt{3+\tan x} + C$$

Test $f\left(\frac{\pi}{4}\right) = 2\sqrt{3+\underset{1}{\cancel{\tan\frac{\pi}{4}}}} + C \overset{\text{set}}{=} -5$

$$2\sqrt{\overset{2}{\cancel{4}}} + C = -5 \Rightarrow C = -9$$

Finally, $\boxed{f(x) = 2\sqrt{3+\tan x} - 9}$

10 $v(t) = \sin t$

(a) $a(t) = v'(t) = \boxed{\cos t}$

and

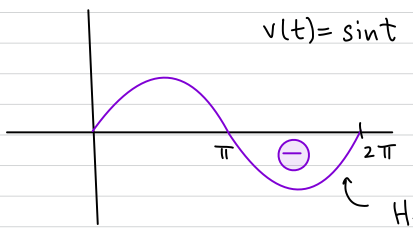
$s(t) = \int v(t)\,dt = \int \sin t\,dt = -\cos t + C$

Test $S(0) = -\cancel{\cos}\overset{1}{}\,0 + C \overset{\text{set}}{=} 2 \Rightarrow C = 3$

$-1 + C = 2$

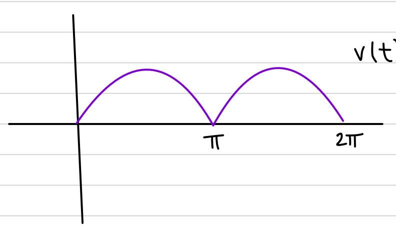
$s(t) = \boxed{-\cos t + 3}$

(b)



Here $v(t)=\sin t$ negative which means Position is decreasing, which means the object is moving to the left for time $t=\pi$ to $t=2\pi$

(c)



(d) Displacement $= \int_0^{2\pi} v(t)\,dt = \boxed{\int_0^{2\pi} \sin t\,dt}$ .. FTC

Total Distance $= \int_0^{2\pi} |v(t)|\,dt = \boxed{\int_0^{2\pi} |\sin t|\,dt}$. Split cases + FTC on each piece